CBDCs, Financial Inclusion, and Optimal Monetary Policy*

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Abstract

This paper studies the interaction between monetary policy and financial inclusion with the introduction of a central bank digital currency (CBDC). Using a New Keynesian two-agent framework with banked and unbanked households, we demonstrate that CBDCs can enhance welfare for the unbanked by serving as an efficient savings tool. A Ramsey optimal policy exercise shows that the CBDC rate typically maintains a constant spread relative to the policy rate. However, a trade-off emerges, as a higher CBDC rate benefits the unbanked but reduces welfare for banked households due to tax redistribution. These findings emphasise the importance of tailoring CBDC design to an economy's level of financial inclusion.

Keywords: central bank digital currency, financial inclusion, optimal monetary policy, Taylor rules, welfare

JEL Classifications: E420, E440, E580

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1 Introduction

Central bank digital currency (CBDC) is a form of digital money, denominated in the national unit of account, which is a direct liability of the central bank.[1] Central banks are actively studying the potential adoption of CBDCs; notable examples include Sweden's E-Krona and China's Digital Currency Electronic Payment. In this paper, we focus on the welfare implications of introducing a retail CBDC. We answer a number of macroeconomic questions on CBDC design: do CBDCs increase welfare of the unbanked through financial inclusion? Do they fundamentally change monetary policy transmission? Should a CBDC be interest bearing, and how should interest rates be optimally set?

We answer these questions using a two-agent New Keynesian (TANK) model with frictions in financial intermediation, and a central bank that sets interest rates on both deposits and the CBDC. Additionally, the two types of households in our model are referred to as the "banked" and "unbanked". banked HH are akin to "unconstrained" households as in, for example, Galí, López-Salido, and Vallés (2007), Bilbiie (2018), and Debortoli and Galí (2017), and operate on their Euler equation due to having access to a non-state-contingent asset, bank deposits. Conversely, the unbanked can only smooth their consumption through real money balances and are subject to a cash-in-advance constraint. We then relax this restriction by allowing both the banked and unbanked households access to an interest-bearing CBDC.

Our analysis proceeds as follows. First, we examine the impact of a CBDC on the transmission of monetary policy. We assume that the central bank follows a standard Taylor rule and that the CBDC interest rate is aligned with the deposit rate. The introduction of a CBDC significantly affects the consumption patterns of unbanked households. By providing access to an interest-bearing savings instrument, the CBDC allows these households to smooth their consumption in response to monetary policy shocks.

1. For more detail on the taxonomy of CBDC designs we refer readers to Auer and Böhme (2020). They discuss many aspects of CBDC design, such as whether the CBDC uses a distributed ledger technology (DLT), is account or token based, or wholesale or retail. In this paper we focus solely on retail CBDCs.

In response to a monetary shock, our findings indicate a trade-off between macroeconomic and financial stability. While output and consumption effects dissipate more rapidly in an economy equipped with a CBDC, the effects of monetary policy shocks on bank equity prices and net worth become more persistent.

Second, we analyse the welfare and distributional effects of CBDC rates in comparison to deposit rates. Our findings indicate that unbanked households benefit when CBDC rates are higher than deposit rates, while banked households experience a welfare decline. This outcome arises from two primary mechanisms: the savings channel and the tax redistribution channel. The savings channel leads to a positive impact on unbanked households as CBDC provides a higher interest rate. The tax redistribution channel further benefits unbanked households, since in our model, CBDC returns are financed by lump-sum taxes equally levied on both banked and unbanked households. Although this may initially seem an unrealistic assumption, the net tax redistribution effects align with what is observed in most taxation systems. Consequently, unbanked households, receiving higher returns from CBDC, become the primary beneficiaries of the tax and CBDC framework.

Third, we perform a Ramsey optimal policy analysis to determine the path of monetary policy that maximises household welfare. The social planner optimises a weighted average of banked and unbanked household welfare using two instruments: the central bank deposit rate and the CBDC interest rate. Our framework tests different CBDC policy regimes, such as adjustable versus fixed rates. We find that when CBDC closely substitutes regular deposits, the optimal policy is to maintain a constant spread between the CBDC rate and the policy rate. In economies with low financial inclusion, welfare gains are mainly due to the introduction of CBDC. In contrast, in economies with higher financial inclusion, the gains stem from optimal monetary policy. Notably, a single-instrument policy where the CBDC rate tracks the policy rate achieves welfare outcomes similar to those of a two-instrument policy.

Related literature. Our work relates to three strands of literature on CBDCs. First, we contribute to a literature understanding the benefits of introducing a CBDC (Chen et al., 2022).[2] Our contribution is to show that the welfare effects depend crucially on the level of financial inclusion, with positive welfare effects on the unbanked through a CBDC increasing savings and acting as a consumption smoothing device.

Second, we contribute to the literature on the implications of CBDC adoption for financial stability (Brunnermeier and Niepelt, 2019; Fernández-Villaverde et al., 2021; Agur, Ari, and Dell'Ariccia, 2022; Andolfatto, 2021; Chiu et al., 2023; Keister and Sanches, 2021; Keister and Monnet, 2022). Important financial stability considerations include studying the competition between bank deposits and CBDCs. For example, Keister and Sanches (2021) determine conditions in which the private sector is disintermediated with CBDC leading to welfare losses. While we introduce a model setup that allows banks to substitute between deposits and CBDC, our model's primary insight is on the re-distributive effects of introducing a CBDC in a two agent framework. Crucially, we find that it can improve welfare and financial inclusion, and reduce consumption inequality.

Finally, we contribute to a growing literature that deals with the closed economy (Burlon et al., 2022; Davoodalhosseini, 2022; Das et al., 2023; Barrdear and Kumhof, 2022; Assenmacher, Bitter, and Ristiniemi, 2023; Abad, Nuño Barrau, and Thomas, 2023) and open economy macroeconomic implications of introducing a CBDC (Ikeda, 2020; Kumhof et al., 2021; Minesso, Mehl, and Stracca, 2022). This includes a discussion of optimal monetary policy and transmission effects, the use of CBDC in a monetarist framework, and the introduction of CBDC on output and the ability to stabilise business cycle fluctuations. Our contribution is to show the transmission of monetary policy and derive the optimal path of interest rates when the central bank controls two instruments: the interest rate on deposits and the CBDC interest rate. The welfare effects on banked and unbanked agents depend crucially on whether the CBDC is in-

2. These studies include the potential for CBDCs to address financial inclusion in emerging market economies such as India and Nigeria, which have a large unbanked population and increasing reliance on digital payments and private payment providers, and theoretical models of financial inclusion in an economy with competition between different types of payments.

terest bearing, and through a Ramsey optimal policy exercise we show that CBDC rates should target a constant spread with respect to the policy rate.

The remainder of the paper is structured as follows. In Section 2 we outline a simple two-period, two-agent endowment economy to clarify our intuition, and examine the welfare implications of introducing a CBDC. In Section 3, we setup the TANK model and state our modelling assumptions. Section 4 examines the effect of introducing a CBDC on monetary policy, including optimal policy exercises for when a social planner can set interest rates on deposits and the CBDC, and examines the welfare implications of alternative rules for targeting the interest rate on the CBDC. Section 5 concludes the paper.

2 Simple Endowment Economy

To highlight the key mechanisms through which digital currency can improve welfare, we consider a simplified two-agent model, where an agent can be of type $i = \{h, u\}$. In this setup, the banked HH (BHH; $i = h$) has access to a first-best risk-free savings device (D), while the unbanked HH (UHH; $i = u$) can save in money balances (M).[3] Each of the agents lives for two periods, receives an initial endowment (y) in the first period, and maximises lifetime utility,

$$u^i = \ln c_1^i + \beta \ln c_2^i,$$

subject to a set of budget constraints for each period.

No digital currency. For the banked, they face the following budget constraints:

$$c_1^h + D = y, \tag{1a}$$

$$c_2^h = RD + \epsilon, \tag{1b}$$

where $R > 1$ is the return on D, and ϵ is a shock that impacts resources in the second period.[4] Conversely, the unbanked face a set of budget constraints and a cash-in-advance

3. We abstract from inflation in this simple setup as we do not discuss considerations in setting interest rates. We include inflation in our TANK framework in Section 3 where we also study optimal monetary policy.

4. In this setup, for simplicity, we do not allow banked agents to hold CBDC as deposits are the first best savings device. We relax this assumption in the TANK model in section 3, where we extend the

(CIA) constraint on their consumption in the second period, and so their constraints are:

$$c_1^u + M = y, \tag{2a}$$

$$c_2^u \leq M + \epsilon, \tag{2b}$$

$$\alpha_M c_2^u \leq M, \tag{2c}$$

where $\alpha_M \in (0, 1]$ is the fraction of consumption that is subject to the cash-in-advance constraint. It is similar to the inverse of the velocity of money. In what follows, we assume $\alpha_M = 1$ for tractability.[5]

Solving for optimal consumption in periods 1 and 2[6] for both households yields the following lifetime consumption ratio:

$$\frac{c_1^h + c_2^h}{c_1^u + c_2^u} = \begin{cases} \frac{\frac{2}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right)}{y + \mathbb{E}[\epsilon]} & \text{if } \mathbb{E}[\epsilon] < 0, \\ \frac{\frac{2}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right)}{y} & \text{if } \mathbb{E}[\epsilon] \geq 0 \end{cases} \tag{3}$$

Figure 1 plots the consumption ratio (3) with respect to the expected value of the shock.[7] As the Figure illustrates, the BHH have higher lifetime consumption than the UHH. These consumption gains are increasing in the magnitude of the income shock. Deposits of the BHH are countercyclical with respect to the income shock: the banked save in anticipation of a negative income shock, and reduce savings in anticipation of positive income shocks, enabling them to better smooth consumption. In contrast, the unbanked do not have access to an interest-bearing consumption smoothing device. This suggests that they are more adversely exposed by negative income shocks. For positive income shocks ($\mathbb{E}[\epsilon] > 0$), we note that the UHH cannot increase consumption in period 2 as they are bounded by the cash-in-advance constraint. Therefore the ratio of lifetime consumption of the banked to unbanked generates a linear relationship with respect to positive expected income shocks.

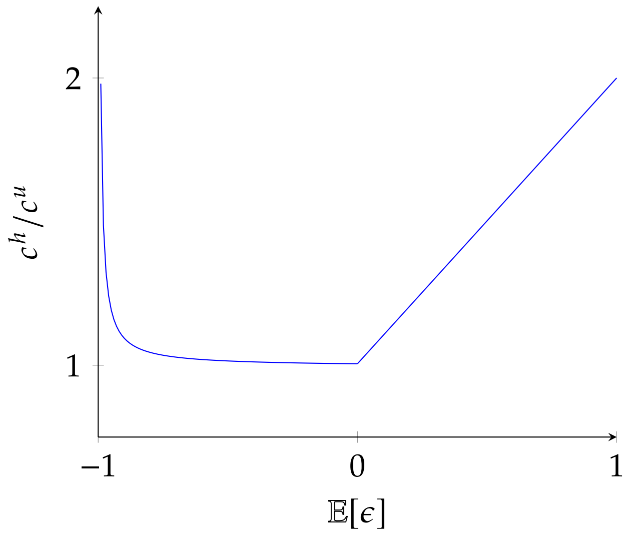
framework to allow banked agents to hold both deposits and CBDC.

5. We check that the results for consumption are qualitatively similar for different values of $\alpha_M < 1$.

6. See Appendix A.1 for details.

7. The expected value of the income shock can be written as $\mathbb{E}[\epsilon] = 1 - 2p$, where p is the probability of a negative realisation of the shock. Therefore for $p \in [0, 1]$, the range of our income shock is $[-1, 1]$.

Figure 1: **Consumption ratios: Banked to unbanked without digital currencies**



Note: Vertical axis: Lifetime consumption ratios of the banked relative to the unbanked HH. Horizontal axis: Period 2 resource shock. For calibration, $\beta = 0.99$ and $y = 1$ and $R = 1/\beta$.

With digital currency. Now assume that the unbanked have access to digital currency (DC) which is an interest bearing savings device that pays out $R^{DC} \leq R$ upon maturity. Their set of budget constraints are now:[8]

$$c_1^u + M + DC = y, \tag{4a}$$

$$c_2^u \leq R^{DC} DC + M + \epsilon, \tag{4b}$$

$$\alpha_M c_2^u \leq M, \tag{4c}$$

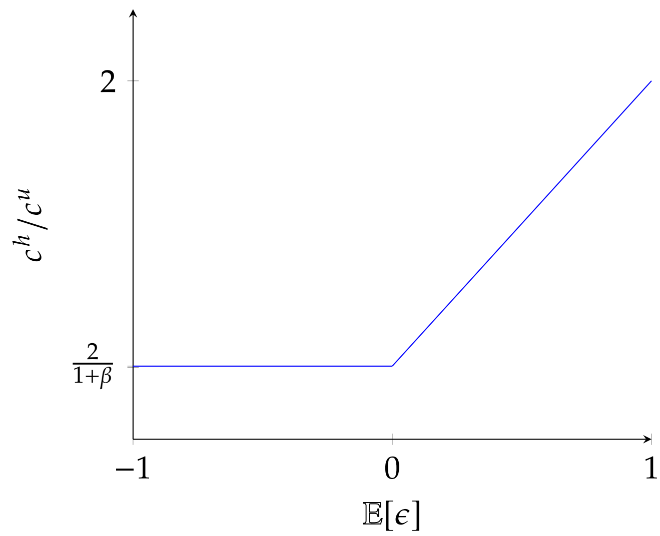
We can analyse the implications of the introduction of digital currency for household lifetime consumption. Repeating the previous exercise, we solve for optimal consumption quantities for the households to express the consumption of the banked to unbanked with digital currencies:

$$\frac{c_1^h + c_2^h}{c_{1,DC}^u + c_{2,DC}^u} = \begin{cases} \frac{2}{1+\beta} & \text{if } \mathbb{E}[\epsilon] < 0, \\ \frac{\frac{2}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right)}{y} & \text{if } \mathbb{E}[\epsilon] \geq 0. \end{cases} \tag{5}$$

Figure 2 plots the consumption ratio (5) with respect to the expected value of the income shock. Introducing digital currency makes the unbanked more resilient with respect to the anticipation of a negative income shock – particularly for large expected

8. Technically, one can extend the access of DC to the banked HH. But so long as the returns to D dominate the returns on DC, the banked will choose to hold no digital currency.

Figure 2: **Consumption ratios: Banked to unbanked with digital currencies**



Note: Vertical axis: lifetime consumption ratios of the banked relative to the unbanked HH after introducing a digital currency. Horizontal axis: Period 2 resource shock. For calibration, $\beta = 0.99$ and $y = 1$ and $R^{DC} = 1/\beta$

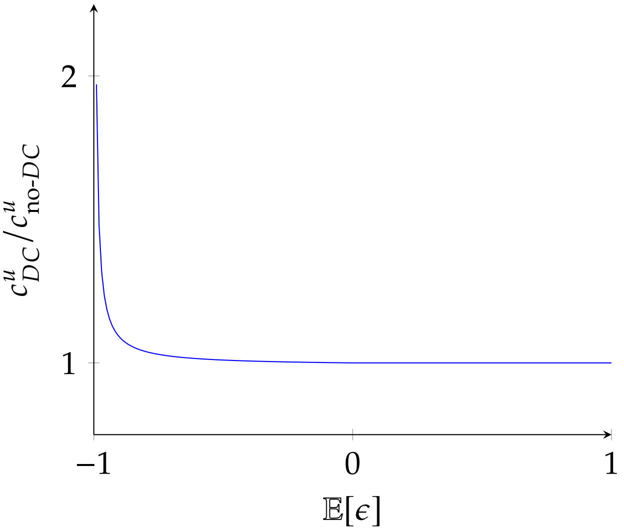
negative shocks – as they now have access to a savings device and can better smooth consumption than with just holding money balances. The ratio of lifetime consumption between the two sets of households is constant with respect to expected negative income shocks ($\mathbb{E}[\epsilon] < 0$). However, for positive anticipated income shocks the digital currency cannot improve the welfare of the UHH. This is due to two factors: (i) the UHH's consumption in period 2 is limited by the CIA constraint, and (ii) we do not allow the unbanked to take a short-position on DC (we require $DC \geq 0$).

Therefore, the ratio of lifetime consumption of the banked to unbanked is identical to the regime with no digital currency in Figure 1 for positive anticipated income shocks; but the unbanked are better off for the case of a large negative anticipated shock. This can be seen by plotting the ratio of lifetime consumption of the unbanked HH under the two regimes – with and without digital currencies, illustrated in Figure 3.

$$\frac{c^u_{1,DC} + c^u_{2,DC}}{c^u_{1,no-DC} + c^u_{2,no-DC}} = \begin{cases} \frac{\frac{2}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R^{DC}}\right)}{y + \mathbb{E}[\epsilon]} & \text{if } \mathbb{E}[\epsilon] < 0, \\ 1 \quad \text{if } \mathbb{E}[\epsilon] \geq 0. \end{cases} \tag{6}$$

In summary, our analysis highlights one channel of welfare improvement associated with introduction of digital currency. If the digital currency is interest bearing,

Figure 3: **Consumption ratios: Unbanked with and without digital currencies**



Note: Vertical axis: lifetime consumption ratios. Horizontal axis: Period 2 resource shock. For calibration, $\beta = 0.99$ and $y = 1$ and $R^{DC} = 1/\beta$.

it is a more efficient savings device than money. It allows the unbanked to engage in more efficient consumption smoothing, particularly providing better insurance against anticipated negative income shocks.

While our simple model sheds light on the role of financial inclusion, this framework is limited as we cannot study: (i) the role of monetary policy, and (ii) whether it is optimal for the interest rate on digital currency to track movements in the policy rate. We now turn to these policy questions in Section 3 by embedding the two-agent framework in a New Keynesian model with digital currency access to both banked and unbanked HH.

3 Two-Agent New Keynesian Model with Central Bank Digital Currency

In this section, we present a two-agent New Keynesian (TANK) model as in Debortoli and Galí (2017, 2022) and Bilbiie (2018). Notably, our model features a banking sector accompanied with credit frictions (Gertler and Karadi, 2011; Gertler and Kiyotaki, 2010). In this framework, a fixed fraction of the banked HH are bankers, which allows us to maintain a representative setup of the household sector. Banked HH hold claims on CBDC and deposits. Deposits are denominated in fiat currency and held at banks. Banked HH may also directly invest in firms by purchasing equity holdings. Banks convert deposits into credit, facilitating loans to firms who acquire capital for the means of production, as in Gertler and Kiyotaki (2010, 2015). Unbanked HH are still limited to money holdings and CBDCs.

3.1 Production

The supply side of the economy is standard. Final goods are produced by perfectly competitive firms that use labour and capital to produce their output.[9] They also have access to bank loans, and conditional on being able to take out a loan, they do not face any financial frictions. These firms pay back the crediting banks in full via profits. Meanwhile, capital goods are produced by perfectly competitive firms, which are owned by the collective household.

Capital good firms. We assume that capital goods are produced by perfectly competitive firms, and that the aggregate capital stock grows according to the following law of motion:

$$K_t = I_t + (1-\delta)K_{t-1}, \tag{7}$$

where I_t is investment and $\delta \in (0,1)$ is the depreciation rate.

The objective of the capital good producing firm is to choose I_t to maximise revenue, $Q_t I_t$. Thus, the representative capital good producing firm's objective function is:

$$\max_{I_t} \left\{ Q_t I_t - I_t - \Phi\left(\frac{I_t}{I}\right) I_t \right\},$$

where $\Phi(\cdot)$ are investment adjustment costs and are defined as:

$$\Phi\left(\frac{I_t}{I}\right) = \frac{\kappa_I}{2}\left(\frac{I_t}{I} - 1\right)^2,$$

with $\Phi(1) = \Phi'(1) = 0$ and $\Phi''(\cdot) > 0$.

Intermediate goods producers. The continuum of intermediate good producers are normalised to have a mass of unity. A typical intermediate firm produces output y_t

9. We relegate the discussion of final good firms to the Appendix A.2.1 as it is standard.

according to a constant returns to scale technology in capital k_t and labour l_t with a common productivity shock A_t

$$y_t = A_t k_{t-1}^{\alpha} l_t^{1-\alpha}.$$

The problem for an intermediate firm is to minimise costs subject to their production constraint, where the demand for their output is given by the standard index:

$$y_t = \left(\frac{p_t}{P_t}\right)^{-\epsilon} Y_t.$$

This yields the minimised unit cost of production:

$$MC_t = \frac{1}{A_t}\left(\frac{z_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}. \tag{8}$$

The price-setting problem of a firm is set up à la Rotemberg (1982) where it maximises the net present value of profits,

$$\mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}^h \left[\left(\frac{p_{t+s}}{P_{t+s}}(1-\tau) - MC_{t+s}\right) y_{t+s} - \frac{\kappa}{2}\left(\frac{p_{t+s}}{p_{t-1+s}} - 1\right)^2 Y_{t+s}\right],$$

by optimally choosing p_t, and where κ denotes a price adjustment cost parameter for the firms.

Evaluating at the symmetric equilibrium where intermediate firms optimally price their output at $p_t = P_t, \forall i$, yields the standard New Keynesian Phillips curve (NKPC):

$$\pi_t(\pi_t - 1) = \frac{\epsilon - 1}{\kappa}(\mathcal{M}_t MC_t + \tau - 1) + \mathbb{E}_t \Lambda_{t,t+1}^h (\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}, \tag{9}$$

where $\mathcal{M}_t$ is the representative intermediate firm's markup.

Also, under the symmetric equilibrium we can express output as:

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}. \tag{10}$$

As noted above, there is a distortion arising from monopolistic competition among intermediate firms. We assume that there is a lump-sum subsidy to offset this distortion, τ. From (9), we see that the policymaker chooses a subsidy such that the markup over marginal cost is offset in the deterministic steady state:[10]

$$\tau = -\frac{1}{\epsilon - 1}$$

which guarantees a non-distorted steady-state. Hereinafter, we abstract from distorted

10. Note that this assumes that steady state inflation is net-zero, i.e., $\pi = 1$.

steady states and only consider the efficient steady state. Our choice to model nominal rigidity following Rotemberg pricing should not alter our welfare analysis in Section 4. As noted by Nisticò (2007) and Ascari and Rossi (2012), up to a second order approximation and provided that the steady state is efficient, models under both Calvo and Rotemberg pricing imply the same welfare costs of inflation. Therefore, a welfare-maximising social planner would prescribe the same optimal policy across the two regimes.

3.2 Households and Workers

The representative household contains a continuum of individuals, normalised to 1, each of which are of type $i \in \{h, u\}$. Bankers and banked workers $(i = h)$ share a perfect insurance scheme, such that they each consume the same amount of real output. However, unbanked workers $(i = u)$ are not part of this insurance scheme, and so their consumption volumes are different from bankers and workers. Similar to before in Section 2, we define Γ_h as the proportion of the BHH and bankers, and the UHH are of proportion $\Gamma_u = 1 - \Gamma_h$.

We endogenise labour supply decisions on the part of households, and so the BHH maximises the present value discounted sum of utility:[11]

$$\mathbb{V}_t^h = \max_{\{C_{t+s}^h, L_{t+s}^h, D_{t+s}, K_{t+s}^h, DC_{t+s}^h\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \Xi_{t+s} \ln\left(C_{t+s}^h - \zeta_0^h \frac{(L_{t+s}^h)^{1+\zeta}}{1+\zeta} \right), \tag{11}$$

subject to their period budget constraint:

$$\begin{aligned} &C_t^h + D_t + Q_t K_t^h + \chi_t^h + DC_t^h + \chi_t^{DC,h} + T_t^h \\ &\qquad = w_t L_t^h + \Pi_t + R_t^k Q_{t-1} K_{t-1}^h + \frac{R_{t-1} D_{t-1} + R_{t-1}^{DC} DC_{t-1}^h}{\pi_t}, \end{aligned} \tag{12}$$

where w_t are real wages, L_t^i is labour supply, $R_t^k = (z_t^k + (1-\delta)Q_t)/Q_{t-1}$ is the gross return on equity or capital, ζ is the inverse-Frisch elasticity of labour supply, ζ_0^i is a relative labour supply parameter, K_t^h are equity holdings in firms by the BHH, χ_t^h are the costs of equity acquisitions incurred by the BHH, $\chi_t^{DC,i}$ are digital currency man-

11. We make use of Greenwood–Hercowitz–Huffman preferences for both the BHH and UHH to eliminate the income effect on an agent's labour supply decision. Additionally, it allows us to develop a tractable analytical solution for the model steady state.

agement costs,[12] T_t^i are lump-sum taxes, Q_t is the price of equity/capital, and Π_t are distribution of profits due to the ownership of banks and firms. There is a shock to agents' preferences, Ξ_t, and it is given by:

$$\Xi_{t+s} = \begin{cases} e^{\xi_1}e^{\xi_2}...e^{\xi_s} & \text{for } s \geq 1, \\ 1 & \text{for } s = 0, \end{cases}$$

where ξ_t is a preference (demand) shock given by an AR(1) process. We also note that $\Lambda_{t,t+s}^h$ is the BHH stochastic discount factor (SDF):

$$\Lambda_{t,t+s}^h \equiv \beta^s \mathbb{E}_t \frac{\Xi_{t+s}\lambda_{t+s}^h}{\lambda_t^h}, \tag{13}$$

where λ_t^h is the marginal utility of consumption for the BHH.

One distinction between banked workers and bankers purchasing equity in firms is the assumption that the worker pays an efficiency cost, χ_t^h, when they adjust their equity holdings. We assume the following functional form for χ_t^h:

$$\chi_t^h = \frac{\varkappa^h}{2}\left(\frac{K_t^h}{K_t}\right)^2 \Gamma_h K_t. \tag{14}$$

Meanwhile, the UHH maximises the present discounted sum of per-period utilities given by:

$$\mathbb{V}_t^u = \max_{\{C_{t+s}^u, L_{t+s}^u, M_{t+s}, DC_{t+s}^u\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \Xi_{t+s} \ln\left(C_t^u - \zeta_0^u \frac{(L_t^u)^{1+\zeta}}{1+\zeta}\right), \tag{15}$$

subject to its budget constraint,

$$C_t^u + M_t + \chi_t^M + DC_t^u + \chi_t^{DC,u} + T_t^u = w_t L_t^u + \frac{M_{t-1} + R_{t-1}^{DC} DC_{t-1}^u}{\pi_t}, \tag{16}$$

and the CIA constraint,

$$\alpha_M C_t^u \leq \frac{M_{t-1}}{\pi_t}. \tag{17}$$

12. The digital currency management costs for household of type i are:

$$\chi_t^{DC,i} = \frac{\varkappa^{DC}}{2}\left(\frac{DC_t^i}{\widetilde{DC}^i}\right)^2, \quad i \in \{h, u\},$$

where $\widetilde{DC}^i$ are target digital currency balances, calibrated in the baseline case such that aggregate holding of digital currencies is one-third of output. Alternatively, we could assume a non-pecuniary motive for holding digital currency that would manifest as an additional term of the same form in the household utility function. This setup would imply the same first-order conditions.

3.3 Banks

Bankers are indexed on the continuum $j \in [0,1]$. Among the population of bankers, each j-th banker owns and operates their own bank which has a continuation probability given by σ_b. A banker will facilitate financial services between households and firms by providing loans to firms in the form of equity, k_t^b, funded by deposits, d_t, and their own net worth, n_t.

As is standard in the literature, bankers face a balance sheet constraint:

$$Q_t k_t^b = d_t + n_t, \tag{18}$$

and a flow of funds constraint:

$$n_t = R_t^k Q_{t-1} k_{t-1}^b - \frac{R_{t-1}}{\pi_t} d_{t-1}, \tag{19}$$

where net worth is the difference between gross return on assets and liabilities. Note that for the case of a new banker, the net worth is the startup fund given by the collective household by fraction γ_b:

$$n_t = \gamma_b R_t^k Q_{t-1} k_{t-1}.$$

The objective of a banker is to maximise franchise value, $\mathbb{V}_t^b$, which is the expected present discount value of terminal wealth:

$$\mathbb{V}_t^b = \mathbb{E}_t \left[\sum_{s=1}^{\infty} \Lambda_{t,t+s}^h \sigma_b^{s-1} (1 - \sigma_b) n_{t+s} \right]. \tag{20}$$

A financial friction in line with Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) is used to limit the banker's ability to raise funds from depositors, whereby the banker faces a moral hazard problem: the banker can either abscond with the funds they have raised from depositors, or the banker can operate honestly and pay out their obligations. Absconding is costly, however, and so the banker can only divert a fraction $\theta^b > 0$ of assets they have accumulated.[13] Thus, bankers face the following incentive compatibility constraint:

$$\mathbb{V}_t^b \geq \theta^b Q_t k_t^b. \tag{21}$$

The problem of the banker consists of maximising (20) subject to the balance sheet

13. It is assumed that the depositors act rationally and that no rational depositor will supply funds to the bank if they clearly have an incentive to abscond.

constraint (18), the evolution of net worth (19), and the incentive compatibility constraint (21).

Since $\mathbb{V}_t^b$ is the franchise value of the bank, which we can interpret as a "market value", we can divide $\mathbb{V}_t^b$ by the bank's net worth to obtain a Tobin's Q ratio for the bank denoted by ψ_t:

$$\psi_t \equiv \frac{\mathbb{V}_t^b}{n_t} = \mathbb{E}_t \Lambda_{t,t+1}^h (1 - \sigma_b + \sigma_b \psi_{t+1}) \frac{n_{t+1}}{n_t}. \tag{22}$$

We define ϕ_t as the maximum feasible asset to net worth ratio, or, rather, the leverage ratio of a bank:

$$\phi_t = \frac{Q_t k_t^b}{n_t}. \tag{23}$$

Additionally, if we define $\Omega_{t,t+1}$ as the stochastic discount factor of the banker, μ_t as the excess return on capital over fiat currency deposits, and υ_t as the marginal cost of deposits, we can write the banker's problem as the following:

$$\psi_t = \max_{\phi_t} \left\{ \mu_t \phi_t + \upsilon_t \right\}, \tag{24}$$

subject to

$$\psi_t \geq \theta^b \phi_t.$$

Solving this problem yields:

$$\psi_t = \theta^b \phi_t, \tag{25}$$

$$\phi_t = \frac{\upsilon_t}{\theta^b - \mu_t}, \tag{26}$$

where:

$$\mu_t = \mathbb{E}_t \Omega_{t,t+1} \left(R_{t+1}^k - \frac{R_t}{\pi_{t+1}} \right), \tag{27}$$

$$\upsilon_t = \mathbb{E}_t \Omega_{t,t+1} \frac{R_t}{\pi_{t+1}}, \tag{28}$$

$$\Omega_{t,t+1} = \Lambda_{t,t+1}^h (1 - \sigma_b + \sigma_b \psi_{t+1}). \tag{29}$$

For a complete solution of the banker, please refer to Appendix A.2.3 and A.2.4.

3.4 Fiscal and Monetary Policy

We assume that the government operates a balanced budget:

$$\frac{R_{t-1}^{DC}}{\pi_t}DC_{t-1} + \frac{M_{t-1}}{\pi_t} = \tau Y_t + \Gamma_h T_t^h + \Gamma_u T_t^u + DC_t + M_t, \tag{30}$$

where it levies taxes to cover the producer subsidy to address the distortions arising from monopolistic competition, money balances, and digital currencies. Our budget constraint allows for money and digital currency to be a liability of the central bank, and is consistent with other studies that model the issuance of CBDC (Barrdear and Kumhof, 2022; Kumhof et al., 2021).

Meanwhile, the central bank is assumed to operate an inertial Taylor rule for the nominal interest rate:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_Y}\right]^{1-\rho_R}\exp(\varepsilon_t^R), \tag{31}$$

where variables without time subscripts denote steady state values. Additionally, we assume that the central bank sets the nominal return on digital currency one-for-one in line with the nominal interest rate on deposits:

$$R_t^{DC} = R_t. \tag{32}$$

We explore the implications of alternative rules on model dynamics and welfare in Section 4.

3.5 Market Equilibrium

Aggregate consumption, labour supply, and digital currency holdings by the BHH and UHH are given as:

$$C_t = \Gamma_h C_t^h + \Gamma_u C_t^u, \tag{33}$$

$$L_t = \Gamma_h L_t^h + \Gamma_u L_t^u, \tag{34}$$

$$DC_t = \Gamma_h DC_t^h + \Gamma_u DC_t^u. \tag{35}$$

Then define ω_t as the consumption inequality factor, as in Debortoli and Galí (2017), between the banked and unbanked HH:

$$\omega_t = 1 - \frac{C_t^u}{C_t^h}. \tag{36}$$

This will allow us to track consumption inequality between the two types of household. Increases (decreases) in ω_t follow from banked HH consuming a larger (smaller) share of aggregate consumption.

The aggregate resource constraint of the economy is:

$$Y_t = C_t + \left[1 + \Phi\left(\frac{I_t}{I}\right)\right] I_t + \frac{\kappa}{2}(\pi_t - 1)^2 Y_t + \Gamma_h(\chi_t^h + \chi_t^{DC,h}) + \Gamma_u(\chi_t^M + \chi_t^{DC,u}), \quad (37)$$

with aggregate capital being given by:

$$K_t = \Gamma_h(K_t^h + K_t^b). \quad (38)$$

Aggregate net worth of the bank is given by:

$$N_t = \sigma_b \left(R_t^k Q_{t-1} K_{t-1}^b - \frac{R_{t-1}}{\pi_t} D_{t-1}\right) + \gamma_b R_t^k Q_{t-1} \frac{K_{t-1}}{\Gamma_h}, \quad (39)$$

and the aggregate balance sheet of the bank is given by the following equations:

$$Q_t K_t^b = \phi_t N_t, \quad (40)$$

$$Q_t K_t^b = D_t + N_t. \quad (41)$$

Finally, the stationary AR(1) processes for TFP, markup, and preference shocks are given by:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A, \quad (42)$$

$$\mathcal{M}_t = (1 - \rho_M)\mathcal{M} + \rho_M \mathcal{M}_{t-1} + \varepsilon_t^M, \quad (43)$$

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t^\xi \quad (44)$$

A competitive equilibrium is a set of eight prices, $\{ MC_t, R_t, R_t^{DC}, R_t^k, \pi_t, Q_t, w_t, z_t^k \}$, nineteen quantity variables, $\{ C_t, C_t^h, C_t^u, D_t, DC_t, DC_t^h, DC_t^u, I_t, K_t, K_t^b, K_t^h, L_t, L_t^h, L_t^u, M_t, N_t, T_t^h, T_t^u, Y_t \}$, four bank variables, $\{ \psi_t, \phi_t, \mu_t, \upsilon_t \}$, and three exogenous variables, $\{ A_t, \xi_t, \mathcal{M}_t \}$, that satisfies 34 equations. For a complete list of the equilibrium conditions please refer to Appendix A.2.5. Steady state solutions are provided in Appendix A.2.6 for the baseline TANK model.

3.6 Model Calibration and Steady State Values

We set model parameters, which are found in standard New Keynesian models, in line with the literature. See, for example, Galí (2015), Walsh (2010), and Woodford (2003).

Table 1: **Parameter values**

θ^b	0.399	Banker absconding ratio
σ_b	0.940	Survival probability
γ^b	0.005	Fraction of total assets inherited by new banks
$DC/4Y$	1/3	DC to Output
β	0.990	Discount rate
ζ	0.333	Inverse-Frisch elasticity
ζ_0^h	3.050	Labour supply disutility
$\varkappa^h$	0.020	Cost parameter of direct finance
Γ_h	0.750	Proportion of BHH
α_M	1	Inverse velocity of money
ϕ_M	0.010	Money adjustment cost parameter
$\varkappa^{DC}$	0.001	Digital currency adjustment cost parameter
α	0.333	Capital share of output
δ	0.025	Depreciation rate
ϵ	10	Elasticity of demand
κ_I	2/3	Investment adjustment cost
θ	0.750	Calvo parameter
τ	0.111	Producer subsidy
$\mathcal{M}$	1.111	Markup
ϕ_π	2	Taylor rule inflation coefficient
ϕ_Y	0.100	Taylor rule output coefficient
ρ_b	0.850	AR(1) coefficient for demand shock
ρ_A	0.850	AR(1) coefficient for TFP shock
ρ_M	0.850	AR(1) coefficient for markup shock
ρ_R	0.550	Taylor rule persistence
σ^A	0.5%	TFP std dev
σ^D	0.1%	Demand shock std dev
σ^M	1%	Markup shock std dev
σ^R	0.5%	MP shock std dev

Parameter values are provided in Table 1.

Model parameters that are not standard, particularly the bank parameters, are set based on Akinci and Queralto (2022). For example, a banker's survival rate, σ_b, is chosen so that the annual dividend payout is a share of $4 \times (1 - \sigma_b) = 0.24$ of net worth. The banker absconding ratio, θ^b; the banker management cost of digital currencies, $\varkappa^b$; and the fraction of total assets inherited by new bankers, γ^b, are chosen so that in steady state the bank leverage ratio is approximately 4 and that the share of equity financed by bank finance is approximately 0.70. Furthermore, parameters pertaining to adjustment costs of money balances, ϕ_M, and of CBDCs, $\varkappa^{DC}$, are calibrated such that digital currency is more easily adjustable than money balances and deposits are the first-best transactions and savings vehicle. Our results are robust to different calibrations of these parameters as long as $0 < \varkappa^{DC} < \phi_M$. We calibrate $\widetilde{DC}$ such that CBDC to output ratio is approximately one third, which is similar to the baseline calibration in (Barrdear and Kumhof, 2022; Kumhof et al., 2021), and implies the ratio of CBDC to the sum of CBDC and deposits of approximately 14%, similar to Assenmacher, Bitter, and Ristiniemi (2023).

Finally, we set the parameters pertaining to monetary policy, namely the sensitivity of nominal interest rates to inflation, ϕ_π, the sensitivity of nominal interest rates to the output gap, ϕ_Y, and the interest rate smoothing parameter, ρ_R, in line with Guerrieri and Iacoviello (2015).

We assume the persistence of our exogenous AR(1) processes to be 0.85 per quarter. The standard deviations of shocks are set at 0.5% per quarter for TFP, and 0.1% for the cost-push, preference, and monetary policy shocks, unless stated otherwise. For example, innovations to shocks are set at 1% when plotting impulse response functions.

4 Dynamics and Welfare Implications

4.1 Impulse Responses to a Monetary Policy shock

Figure 4 presents impulse responses to a 1% (annualised) monetary policy tightening with the Taylor rule (31) and $R_t = R_t^{DC}$.[14] We plot impulse responses for two alternative regimes: a CBDC-equipped economy as described in Section 3 (red dashed line) and an economy with no CBDCs (blue line).

Under the no-DC regime, the monetary policy tightening has standard responses for the real economy: output, consumption, and the marginal cost decline in response to the increase in the real interest rate. Here consumption inequality (ω) initially improves as the unbanked HH benefits from the deflationary pressure in the economy, increasing real money balances.[15] The impact on financial variables are also in line

14. A full set of IRFs can be found in Appendix A.2.7, and Figure 13 plots the impulse responses to the monetary policy tightening for a broader set of model variables.

15. For brevity, we avoid plotting wages, the banked HH labour supply, and the banked HH consump-

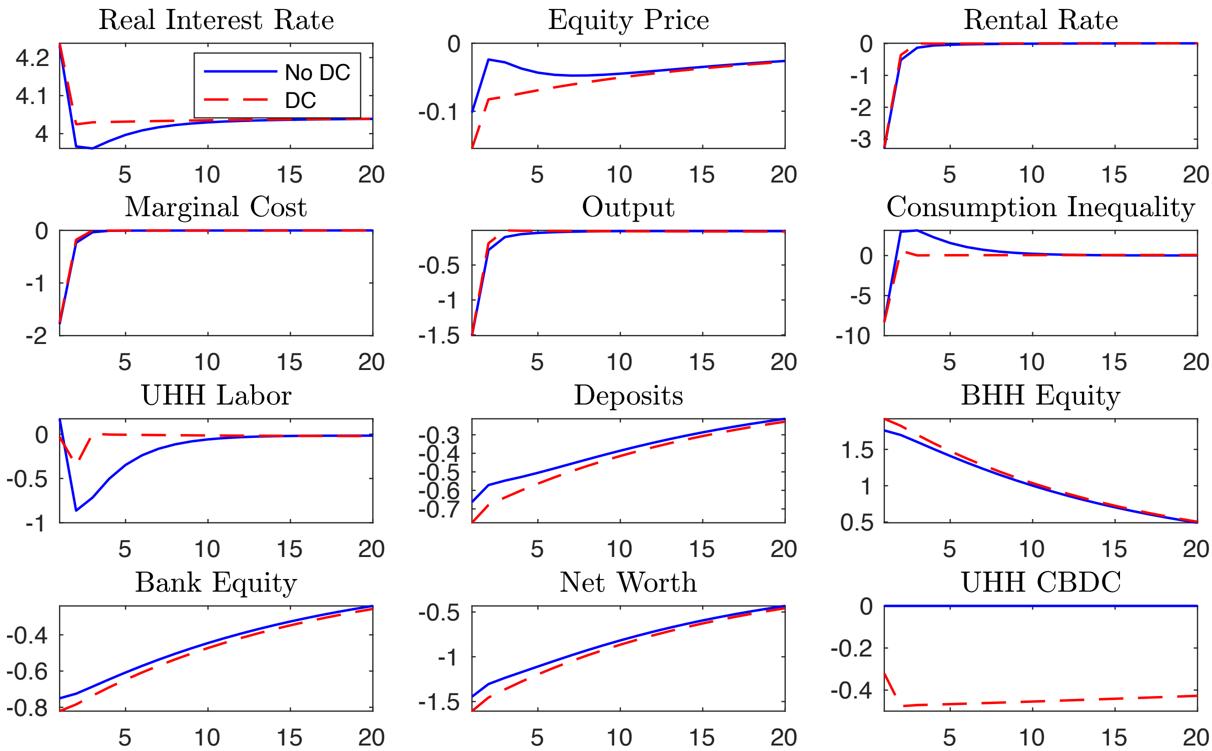
with DSGE models with financial intermediation (see for example Gertler and Kiyotaki (2010) and Gertler and Karadi (2011)): A small decline in the price of equity leads to a decline in bank intermediation as bank equity, deposits, and net worth shrinks, affecting the real economy via the financial accelerator mechanism (Kiyotaki and Moore, 1997; Bernanke, Gertler, and Gilchrist, 1999).

In the presence of a CBDC, the dynamics of the economy differ from those in a no-CBDC economy, largely due to the unbanked HH being able to better smooth their consumption and monetary policy increasing in efficacy. The intuition behind these differences is as follows: In the no-CBDC economy, an initial increase in the real interest rate and a reduction in consumption inequality are reversed in the subsequent period as the unbanked HH consumption drops sharply, and the real interest rate overshoots below its steady-state level. While this is good for investment (Q, the price of equity, falls by less in the no-DC economy), aggregate capital, and bank variables; this overshooting of the real interest rate occurs because the central bank acts to close the output gap. However, when the unbanked HH has access to a CBDC, their consumption becomes significantly less volatile in response to a shock. This not only means that consumption inequality is improved between the banked and unbanked HHs, but also that monetary policy's efficacy is amplified since the interest rate on CBDCs tracks the deposit rates.

In simpler terms, the introduction of a CBDC notably alters the response of unbanked households' consumption. Without access to a CBDC, unbanked households must drastically cut consumption when they rely solely on real money balances as a savings vehicle. The availability of a CBDC allows these households to mitigate the impact of shocks by reducing their savings (as seen in the decline of DC^u), thereby attenuating the drop in consumption. These mechanisms were highlighted in our simple endowment economy in Section 2. By providing an effective savings device, a CBDC mutes the aggregate response of consumption and output, leading to quicker dissipation of monetary shocks compared to a no-CBDC economy. For banked households,

tion as they are highly correlated with output due to the specification of GHH preferences and operating on a standard consumption Euler equation.

Figure 4: **IRFs to a 1% annualised monetary policy (tightening) shock**



Note: Figure plots impulse responses of model variables with respect to a 1% annualised innovation to the Nominal Interest Rate. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π) and Nominal Interest Rates (R) which are expressed as annualised net rates.

the introduction of a CBDC has little effect on their consumption response to monetary policy shocks, as they already have access to bank deposits, and do not significantly adjust their CBDC holdings in response to the shock. Thus, the introduction of a CBDC dampens the transmission of monetary policy to unbanked consumption.[16]

As mentioned, we note a policy trade-off between macroeconomic and financial stability in response to monetary policy shocks. Although the CBDC-equipped economy leads to enhanced macroeconomic stabilisation of output and consumption, and the return on capital, bank-related variables—such as net worth and equity—suffer more persistent declines, influenced by equity price dynamics and the financial accelerator effect.

To explore this further, we simulate the two economies to capture the conditional standard deviations of selected macroeconomic and financial variables when the model

16. IRFs for TFP, cost-push, and demand shocks are provided in Appendix A.2.7.

Table 2: Model simulated standard deviations (%)

	no-CBDC	CBDC
Output, Y	2.39	2.51
Inflation, π	1.51	1.44
Nominal Interest Rate, R	2.04	1.90
Net Worth, N	6.68	6.57
Bank Leverage, ϕ	1.81	1.73

Note: Standard deviations are conditional standard deviations based on model simulations. The model is solved and simulated via second-order perturbation about the deterministic steady state. Inflation and the Nominal Interest Rate are annualised.

is subject to all shocks. Table 2 summaries these results. An interesting trade-off emerges: while inflation, the policy rate, and bank financial variables are more stable with the introduction of a CBDC, output volatility is slightly increased relative to the no-CBDC economy. This is due to changes in consumption patterns of the unbanked HH when it has access to CBDCs. Without CBDCs its consumption is procyclical with respect to cost-push, demand, and monetary policy shocks, and counter-cyclical with respect to TFP shocks. However, with CBDCs unbanked HH consumption in response to TFP shocks flips and becomes procyclical, and mildly counter-cyclical with respect to demand shocks – and recall that its consumption response is attenuated in response to monetary policy shocks. The end result is that the strong procyclicality of unbanked HH consumption in response to TFP shocks dominates and leads to higher volatility of GDP.[17]

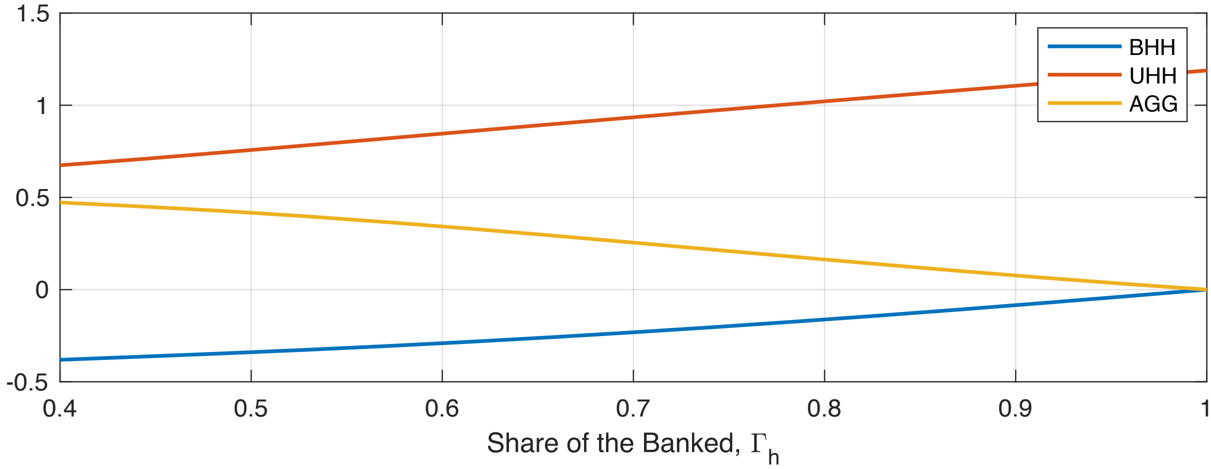
4.2 Welfare Effects

4.2.1 CBDC introduction

Figure 5 evaluates the welfare effects of introducing a CBDC, when the economy is subject to TFP, cost-push, demand, and monetary policy shocks, and monetary policy is conducted according to the Taylor rule (31). We find that the unbanked experience welfare gains in the CBDC-equipped economy. This is due to CBDC offering a rate of remuneration and it being a more efficient savings device than money balances, allowing the unbanked to better insure against adverse shocks.

17. As a robustness check, if we disable TFP shocks in the model, volatility is decreased across the

Figure 5: **Welfare comparison (CBDC regime %ch. over no-CBDC regime)**

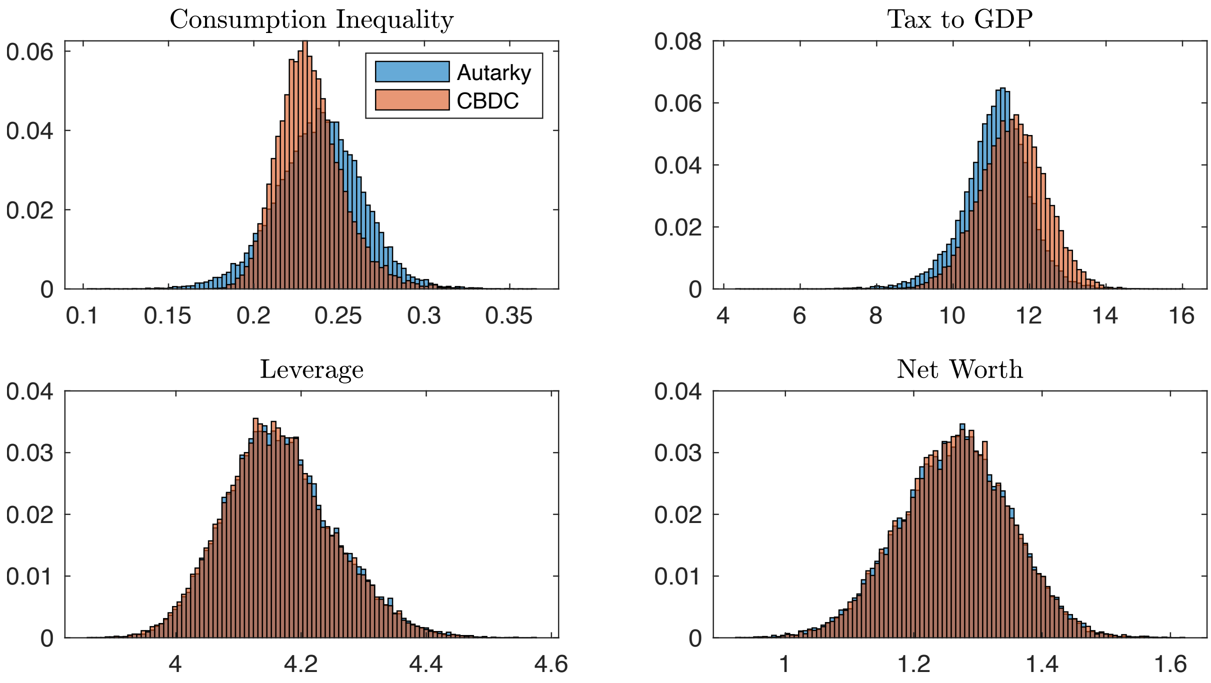


Note: Figure plots welfare for BHH, UHH and aggregate households as a function for the share of the banked population, Γ_h. The welfare is calculated as a per cent change from the regime with no digital currency.

Turning to the banked HH, we find that they experience net negative welfare benefits after introduction of the CBDC. To explain this we note two factors. First, the banked HH face management costs in holding a CBDC relative to bank deposits, and therefore do not gain directly from access to a CBDC as they already have bank deposits – which are a first best transaction and savings device. Second, the banked HH experience net negative welfare losses due to the tax redistribution effects of issuing a CBDC. We can see these effects clearly in Figure 6, which simulates the model economy with the CBDC and no-CBDC (autarky) economies, and where the banked share is $\Gamma_h = 0.75$. In the top panel, we plot the distribution of consumption inequality and the tax-to-GDP ratio. As a CBDC is introduced, we observe a decline in consumption inequality, which we interpret as an increase in the relative consumption of the unbanked HH. The direct consequence of CBDC issuance is an increase in lump-sum taxes levied equally on both households. This is evident based on the rightward shift of the distribution of Tax to GDP. The increased tax burden on banked HH, in conjunction with bank deposits being close substitutes to CBDC, lead to negative net welfare losses for banked HH.

board in the CBDC economy relative to the no-CBDC economy.

Figure 6: **Redistributive effects of CBDCs are not being led by financial disintermediation**



Note: Plot of simulations with 10,000 periods of consumption inequality, tax to GDP, bank leverage and net worth for autarky and CBDC regime. Simulations are subject to TFP, monetary, cost-push and preference shocks in baseline calibration.

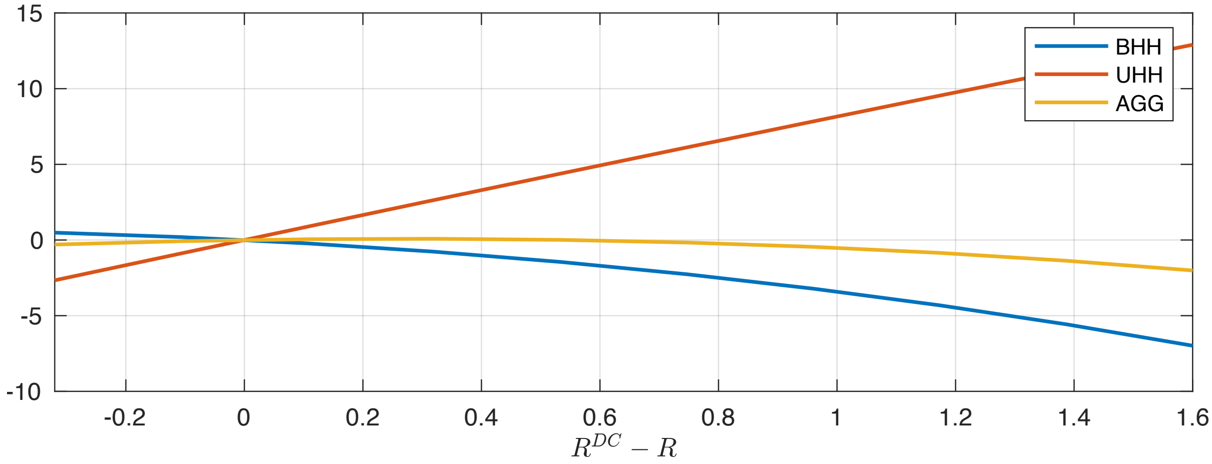
It is important to note that the welfare effects are not driven by disintermediation. The simulations in Figure 6 show that bank leverage and net worth are similar in both the CBDC regime and the no-CBDC regime. Therefore the tax redistribution mechanism is quantitatively more important to explain the welfare effects we find in our model setting.

Turning back to aggregate welfare, we observe net welfare benefits are highest when the economy is primarily unbanked, as shown in Figure 5. As the proportion of the unbanked population declines, the welfare benefits of introducing CBDC tend to zero which suggests a stronger use case of CBDCs in emerging markets with lower degrees of financial inclusion.

4.2.2 Constant Spread Rules

To further illustrate the savings and tax redistribution channels of welfare, we examine the implications of varying levels of CBDC interest rates, measured by the spread

Figure 7: **CBDC economy welfare comparison** (% ch.)



Note: Figure plots relative welfare gains for BHH, UHH, and aggregate households as a function of the spread between the policy rate and the CBDC rate. Note that $\Gamma_h = 0.75$.

$R_t^{DC} - R_t$. We keep the baseline degree of financial inclusion constant ($\Gamma_h = 0.75$) and consider an economy subject to TFP, cost-push, and preference shocks, with monetary policy conducted according to the Taylor rule. Figure 7 depicts the relative welfare gains and losses of agents in a CBDC-equipped economy compared to a benchmark scenario with a zero spread between the CBDC and policy rates ($R_t^{DC} = R_t$). The spread is expressed in annualised percentage form.

Our findings indicate that the unbanked HH benefits more when CBDC rates exceed the policy rate. This is consistent with the savings channel, where the unbanked gain from higher interest rates on CBDC, which provide a buffer against adverse shocks. Conversely, the banked HH experiences a welfare loss when CBDC rates are higher than the policy rate due to the aforementioned tax redistribution effect. As CBDC rates increase, both groups shift toward holding more CBDC, funded by increased lump-sum taxes imposed on all households. For the banked HH, the tax redistribution costs outweigh the benefits of holding digital currency.

In summary, setting the optimal spread between the CBDC and policy rate depends on the level of financial inclusion. Our model suggests that economies with lower financial inclusion and a larger unbanked population should ideally set a higher spread between digital currency rates and policy rates. In contrast, developed economies with a predominantly banked population should set CBDC rates lower than the policy rate. This is consistent with pilot studies in advanced economies with high financial inclusion, such as Sweden's E-Krona, which typically propose a non-interest-bearing currency.

4.3 Optimal Monetary Policy with CBDCs

We now explore the implications for optimal policy, assuming that a policymaker has access to two instruments in order to maximise welfare: nominal interest rates on deposits, R, and nominal interest rates on the CBDC, R^{DC}. More formally, let us state the problem for the welfare maximising policymaker as:

$$\max_{\{R_{t+s}, R^{DC}_{t+s}\}_{s=0}^{\infty}} \mathbb{V}_t = \Gamma_h \mathbb{V}_t^h + \Gamma_u \mathbb{V}_t^u, \tag{45}$$

subject to the entire set of structural equations as set out in Section 3. As CBDC and deposits are imperfect substitutes, the instruments available to the policymaker are not collinear, allowing us to conduct the optimal policy exercise.[18]

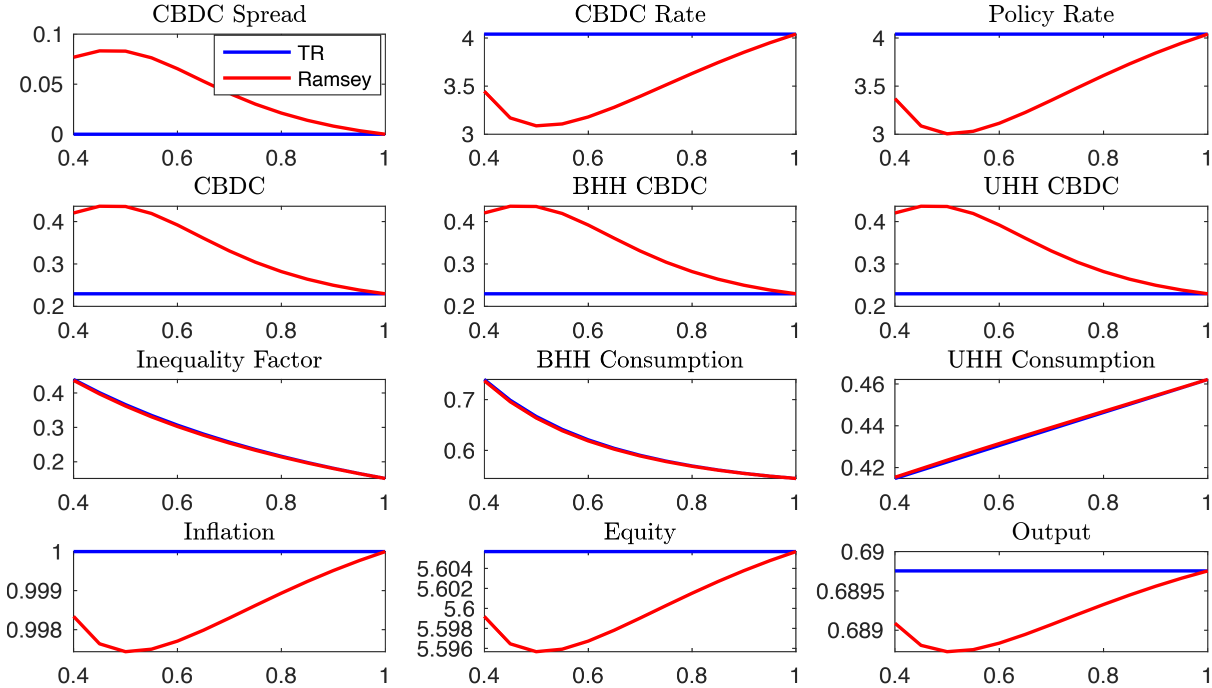
4.3.1 Steady state analysis

The steady-state values implied by the solution to the social planner's problem are shown in Figure 8. The choice of instruments by the Ramsey policymaker leads to a steady state that generally differs from the one under the baseline configuration with a Taylor rule. The presence of unbanked HH subject to a CIA constraint prompts the social planner to select a deflationary steady state. This result is well-documented in the literature, as seen in works by Chari, Christiano, and Kehoe (1991) and Schmitt-Grohé and Uribe (2010). However, since deflation incurs costs through inefficient price adjustments, the policymaker opts for a relatively low level of deflation.

As the share of unbanked HHs converges to zero ($\Gamma_h \to 1$), indicating greater financial inclusion, the model approaches a standard representative agent setup, and the optimal net inflation rate converges to zero, $\pi \to 1$. Furthermore, when financial

18. We argue that R^{DC} is different to R as a Ramsey-instrument in two distinct ways. First, DC is a sub-optimal consumption smoothing instrument to D due to the presence of convex adjustment costs. Secondly, R^{DC} can be set to address consumption inequality and alleviate the CIA constraint of the unbanked, whereas deposit rates cannot be used to address the welfare of the unbanked.

Figure 8: **Steady state values and financial inclusion**



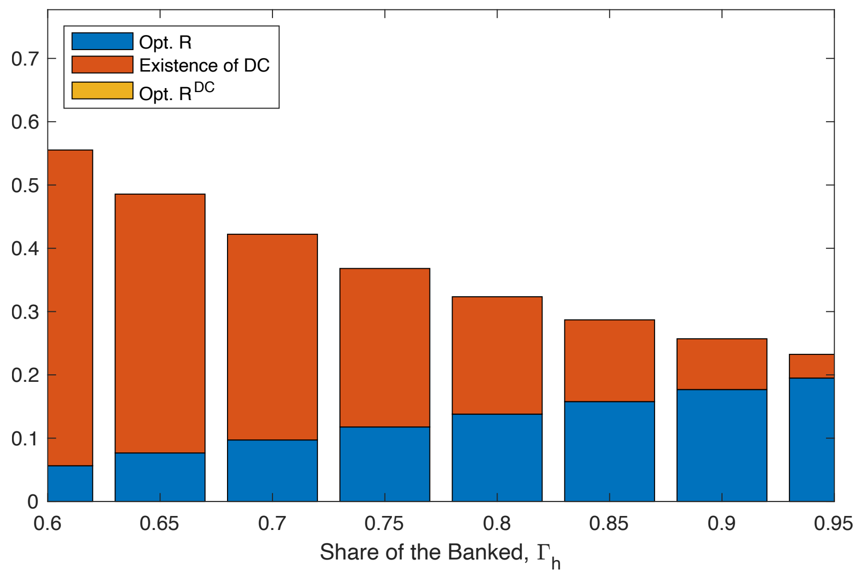
Note: Vertical axis indicates absolute values of variables in steady state, except for π, R, and R^{DC}: these variables are represented as annualised rates. The horizontal axes are values of financial inclusion parameter Γ_h.

inclusion is relatively low ex-ante, the social planner selects higher steady-state CBDC holdings by choosing a larger spread between R^{DC} and R. This occurs because, in maximising the aggregate welfare of the economy as expressed in (45), the social planner aims to redistribute resources from the wealthier banked HH to the unbanked HH, which can be achieved only through interest-bearing CBDC holdings.

4.3.2 Welfare decomposition: Optimal policy and CBDC introduction

The prior welfare exercises in section 4.2 conducted monetary policy with a Taylor rule. Figure 9 shows the decomposition of welfare gains associated with both the introduction of the CBDC and optimal monetary policy. For different levels of the banked population share, we decompose welfare improvements associated with the transition from the no-CBDC economy and a standard Taylor rule, to the CBDC-equipped economy and a Ramsey-optimal monetary policy (two instruments). The model economy is subject to TFP, markup and preference shocks. These welfare gains are associated

Figure 9: **Welfare improvement decomposition**



Note: Vertical axis indicates percent increase in welfare compared to baseline specification without digital currency access.

with: (i) the introduction of CBDCs, (ii) optimal conventional monetary policy, and (iii) optimal R_t^{DC} setting.[19]

We observe that for the economy with low financial inclusion, welfare improvements are mainly associated with the introduction of a CBDC. This is consistent with our earlier findings on how a larger share of the unbanked population increases the welfare gains due to using CBDC as a savings device.

For economies with a higher level of financial inclusion (Γ_h increasing), welfare improvements are due primarily to optimal monetary policy, with the interest rate on CBDC tracking the policy rate. This is intuitive, as a higher share of the banked population means there is a natural amplification of monetary transmission, by changing the response of capital and production through bank balance sheets via a financial accelerator mechanism. The increased importance of monetary policy to stabilise macroeconomic fluctuations increases the gains from conducting optimal monetary policy rela-

19. We compare welfare under the three policy changes to the baseline Taylor-rule regime and no CBDCs. The welfare improvements associated with each regime change do not include cross effects, which are small in magnitude. We approximate all the models around the Ramsey-optimal steady state to ensure that welfare rankings are not spurious, following Benigno and Woodford (2012). This implies steady-state deflation and a spread between R^{DC} and R.

tive to a benchmark Taylor rule.

Evaluating optimal policy design, we observe negligible welfare improvements from optimal policy with one instrument, in which the CBDC rate tracks the policy rate ($R^{DC} = R$), to optimal policy with two instruments, in which the policy rate and the CBDC rate are set independently. This suggests that while an optimal spread is typically non-zero, according to the steady state values of the optimal spread in Figure 8, it leads to quantitatively similar welfare to a rule where the CBDC rate tracks the policy rate.

In summary, the welfare decomposition suggests that gains from introducing a CBDC diminish as financial inclusion increases. Optimal monetary policy is quantitatively similar to a rule in which the rate on CBDC tracks the policy rate. Deviating from this rule results in negligible welfare improvements and is an order of numerical approximation error.

5 Conclusion

This paper analyses the implications of introducing an interest-bearing central bank digital currency (CBDC) on the transmission of monetary policy, distributional and welfare effects, and optimal conduct of monetary policy.

First, the introduction of a CBDC amplifies monetary policy transmission; once a CBDC is introduced, the central bank has an additional instrument to affect the unbanked population which improves monetary policy pass-through. We find that with CBDCs, the volatility of key macroeconomic variables are reduced, albeit with a slight increase in the volatility of GDP. But this is due to increased cyclicality of unbanked household consumption.

Second, banked households are worse-off after a CBDC is introduced while the unbanked are better-off. This is explained by the fact that banked households already have a first-best savings device, bank deposits, while the unbanked gain a more efficient savings device (relative to money balances) and actively use it. Most of the welfare effects go through the tax redistribution channel; as the unbanked use the CBDC more than the banked, they gain more from it, while contributing to its issuance equally.

Third, we find that unbanked households are better-off when the CBDC remuneration rate is set above the deposit or headline policy rate, while banked households are worse off. Furthermore, we find that it is optimal that the CBDC remuneration rate follows the policy rate with a constant positive spread.

Our paper contributes to the ongoing literature exploring the macroeconomic effects of CBDC issuance by using a stylised model prioritising tractability. However, potential areas of research that could be built on our model would be to consider: i) the role of occasionally binding constraints, such as on the bank incentive compatibility constraint and/or the zero lower bound, and ii) the effect of financial disintermediation with "direct CBDCs" on financial stability and macroeconomic performance.

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A Appendix

A.1 Simple Endowment Economy

No digital currency. Assuming that the banked and unbanked face constraints (1) and (2), respectively, and make the simplifying assumption that:

$$\epsilon = \begin{cases} -1 & \text{w.p.} \quad p, \\ 1 & \text{w.p.} \quad 1-p, \end{cases}$$

where $p \in (0,1)$.

Solving the BHH problem for optimal consumption across periods yields

$$c_1^h = \frac{1}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right), \tag{46}$$

$$c_2^h = \frac{\beta}{1+\beta}R\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right), \tag{47}$$

$$D = \frac{\beta}{1+\beta}y - \frac{\mathbb{E}[\epsilon]}{(1+\beta)R}, \tag{48}$$

with the standard consumption Euler equation:

$$c_2^h = \beta R c_1^h.$$

As expected, c_1^h and c_2^h are decreasing in p, while D is increasing in p, highlighting the role of consumption smoothing for the banked HH.

For the UHH, it is clear that the CIA constraint is not binding if $p > \frac{1}{2}$, which yields the following solutions:

$$c_1^u = \frac{1}{1+\beta}\left(y + \mathbb{E}[\epsilon]\right), \tag{49}$$

$$c_2^u = \frac{\beta}{1+\beta}\left(y + \mathbb{E}[\epsilon]\right), \tag{50}$$

$$M = \frac{\beta}{1+\beta}y - \frac{\mathbb{E}[\epsilon]}{1+\beta}, \tag{51}$$

and where their Euler equation is:

$$c_1^u = \beta c_2^u.$$

In the case where $p < \frac{1}{2}$ we have:

$$c_1^u = \frac{1}{1+\beta}y, \tag{52}$$

$$c_2^u = \frac{\beta}{1+\beta}y, \tag{53}$$

$$M = \alpha_M c_2^u. \tag{54}$$

With digital currency. The banked problem remains the same as without digital currency. The unbanked now face constraints in (4), and solving their problem yields the following FOCs:

$$\frac{1}{c_1^u} = \lambda_1,$$

$$\frac{1}{c_2^u} = \lambda_2 + \alpha_M \mu,$$
$$\lambda_1 = \beta \lambda_2 R^{DC},$$
$$\lambda_1 = \beta \lambda_2 + \beta \mu,$$

where λ_t is the period-t marginal utility of consumption and μ is the CIA constraint Lagrangian multiplier. Rearrange the above FOCs, and combine with the fact that for $R^{DC} > 1$ (4c) binds with equality, to get:

$$\frac{1}{c_2^u} = \lambda_2 \left[1 + \alpha_M (R^{DC} - 1)\right],$$
$$c_2^u = \mathcal{S} c_1^u,$$
$$M = \alpha_M c_2^u,$$

where $\mathcal{S} = \beta R^{DC} / \left[1 + \alpha_M (R^{DC} - 1)\right]$ is the marginal rate of transformation of c_1^u and c_2^u – the discounted return on deferring consumption using M and DC. Then write the optimal quantities for the unbanked as:

$$c_1^u = \frac{1}{1+\beta} \left(y + \frac{\mathbb{E}[\epsilon]}{R^{DC}} \right), \tag{55}$$
$$c_2^u = \frac{\mathcal{S}}{1+\beta} \left(y + \frac{\mathbb{E}[\epsilon]}{R^{DC}} \right), \tag{56}$$
$$M = \alpha_M c_2^u, \tag{57}$$
$$DC = \frac{\mathcal{S}(1-\alpha_M)}{(1+\beta)R^{DC}} \left(y + \frac{\mathbb{E}[\epsilon]}{R^{DC}} \right) - \frac{\mathbb{E}[\epsilon]}{R^{DC}}. \tag{58}$$

There is a second case to the problem of the unbanked: when the second period budget constraint does not bind with equality but the CIA does. This yields the following expressions for consumption and digital currency holdings:

$$c_1^u = \frac{\alpha_M}{\alpha_M + \beta} y, \tag{59}$$
$$c_2^u = \frac{\beta}{\alpha_M + \beta} y, \tag{60}$$
$$DC = 0. \tag{61}$$

To understand the two cases, assume for simplicity that $\alpha_M = 1$. This means that (58) simplifies to

$$DC = -\frac{\mathbb{E}[\varepsilon]}{R^{DC}} \tag{62}$$

Since there is a non-negativity constraint on DC, it would imply that the above expression yields a positive balance of DC if and only if $p > \frac{1}{2}$. In other words, if the expected value of the income shock is negative, then an unbanked HH will attempt to save in DC in order to fund its consumption in the second period. If the expected value of the income shock is positive, then the unbanked HH would attempt to take a short position to increase period 2 consumption – which would violate the non-negativity constraint we placed on DC. Hence, in the simplifying case where $\alpha_M = 1$, expected lifetime consumption of the unbanked with and without DC is given by

$$c^u_{\text{w/ }DC} = \begin{cases} y - \frac{1}{R^{DC}} & \text{w.p. } p, \\ y & \text{w.p. } 1-p, \end{cases} \tag{63}$$

$$c^u_{\text{w/o } DC} = \begin{cases} y - 1 & \text{w.p. } p, \\ y & \text{w.p. } 1 - p. \end{cases} \tag{64}$$

A.2 TANK model with Central Bank Digital Currency

A.2.1 Final Good Firms

There is a representative competitive final good producing firm which aggregates a continuum of differentiated intermediate inputs according to a Dixit-Stiglitz aggregator:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{65}$$

Final good firms maximise their profits by selecting how much of each intermediate good to purchase, and so their problem is:

$$\max_{Y_t(i)} P_t Y_t - \int_0^1 P_t Y_t(i) di.$$

Solving for the FOC for a typical intermediate good i is:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon} Y_t. \tag{66}$$

The relative demand for intermediate good i is dependent of i's relative price with ϵ, the price elasticity of demand, and is proportional to aggregate output, Y_t.

From Blanchard and Kiyotaki (1987), we can derive a price index for the aggregate economy:

$$P_t Y_t \equiv \int_0^1 P_t(i) Y_t(i) di.$$

Then, plugging in the demand for good i from (66) we have:

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}.$$

A.2.2 Household Optimisation Problem

The FOCs to the BHH problem are:

$$\lambda^h_t = \frac{1}{C^h_t - \zeta^h_0 \frac{(L^h_t)^{1+\zeta}}{1+\zeta}}, \tag{67}$$

$$w_t = \zeta^h_0 (L^h_t)^{\zeta}, \tag{68}$$

$$1 = \mathbb{E}_t \Lambda^h_{t,t+1} \frac{R_t}{\pi_{t+1}}, \tag{69}$$

$$1 = \mathbb{E}_t \Lambda^h_{t,t+1} \left(\frac{z^k_{t+1} + (1-\delta) Q_{t+1}}{Q_t + \varkappa^h \Gamma_h \left(\frac{K^h_t}{K_t} \right)} \right), \tag{70}$$

$$1 + \varkappa^{DC} \frac{DC^h_t}{\widetilde{DC}^h} = \mathbb{E}_t \Lambda^h_{t,t+1} \frac{R^{DC}_t}{\pi_{t+1}}. \tag{71}$$

The FOCs to the UHH problem are:

$$\lambda_t^u + \alpha_M \mu_t^u = \frac{1}{C_t^u - \zeta_0^u \frac{(L_t^u)^{1+\zeta}}{1+\zeta}}, \tag{72}$$

$$\lambda_t^u w_t = \frac{\zeta_0^u}{C_t^u - \zeta_0^u \frac{(L_t^u)^{1+\zeta}}{1+\zeta}} (L_t^u)^{\zeta}, \tag{73}$$

$$\lambda_t^u \left[1 + \phi_M(M_t - M)\right] = \beta \mathbb{E}_t \xi_{t+1} \left[\frac{\lambda_{t+1}^u + \mu_{t+1}^u}{\pi_{t+1}}\right], \tag{74}$$

$$1 + \varkappa^{DC} \frac{DC_t^u}{\widetilde{DC}^u} = \beta \mathbb{E}_t \xi_{t+1} \frac{\lambda_{t+1}^u}{\lambda_t^u} \frac{R_t^{DC}}{\pi_{t+1}}. \tag{75}$$

A.2.3 Rewriting the Banker's Problem

To setup the problem of the banker as in Section 3.3, first iterate the banker's flow of funds constraint (19) forward by one period, and then divide through by n_t to yield:

$$\frac{n_{t+1}}{n_t} = \frac{\left(z_{t+1}^k + (1-\delta)Q_{t+1}\right)}{Q_t} \frac{Q_t k_t^b}{n_t} - \frac{R_t}{\pi_{t+1}} \frac{d_t}{n_t}.$$

Rearrange the balance sheet constraint (18) to yield the following:

$$\frac{d_t}{n_t} = \phi_t - 1.$$

Substitute this value for d_t/n_t into the expression for n_{t+1}/n_t, and we get:

$$\frac{n_{t+1}}{n_t} = \left(R_{t+1}^k - \frac{R_t}{\pi_{t+1}}\right) \phi_t + \mathbb{E}_t \frac{R_t}{\pi_{t+1}}.$$

Substituting this expression into (22), yields the following:

$$\begin{aligned} \psi_t &= \mathbb{E}_t \Lambda_{t,t+1}^h (1 - \sigma_b + \sigma_b \psi_{t+1}) \left[\left(R_{t+1}^k - \frac{R_t}{\pi_{t+1}}\right) \phi_t + \frac{R_t}{\pi_{t+1}}\right] \\ &= \mu_t \phi_t + \upsilon_t, \end{aligned}$$

which is (24) in the text.

A.2.4 Solving the Banker's Problem

With $\{\mu_t\} > 0$, the banker's incentive compatibility constraint binds with equality, and so we can write the Lagrangian as:

$$\mathcal{L} = \mu_t \phi_t + \upsilon_t + \lambda_t (\psi_t - \theta^b \phi_t),$$

where λ_t is the Lagrangian multiplier. The FOCs are:

$$(1 + \lambda_t)\mu_t = \lambda_t \theta^b, \tag{76}$$

$$\psi_t = \theta^b \phi_t. \tag{77}$$

Substitute (77) into the banker's objective function to yield:

$$\phi_t = \frac{\upsilon_t}{\theta^b - \mu_t}, \tag{78}$$

which is (26) in the text.

A.2.5 Full Set of Equilibrium Conditions

Households.

$$w_t = \zeta_0^h L_t^h \tag{79}$$

$$1 = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{R_t}{\pi_{t+1}} \tag{80}$$

$$1 = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{z_{t+1}^k + (1-\delta)Q_{t+1}}{Q_t + \varkappa^h \Gamma_h \left(\frac{K_t^h}{K_t}\right)} \tag{81}$$

$$1 + \varkappa^{DC} \frac{DC_t^h}{\widetilde{DC}^h} = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{R_t^{DC}}{\pi_{t+1}} \tag{82}$$

$$C_t^u + M_t + \chi_t^M + DC_t^u + \chi_t^{DC,u} + T_t^u = w_t L_t^u + \frac{M_{t-1}}{\pi_t} + \frac{R_{t-1}^{DC}}{\pi_t} DC_{t-1}^u \tag{83}$$

$$\frac{\lambda_t^u}{\lambda_t^u + \alpha_M \mu_t^u} w_t = \zeta_0^u (L_t^u)^\zeta \tag{84}$$

$$\lambda_t^u + \alpha_M \mu_t^u = \frac{1}{C_t^u - \zeta_0^u \frac{(L_t^u)^{1+\zeta}}{1+\zeta}} \tag{85}$$

$$\beta \mathbb{E}_t \xi_{t+1} \frac{\lambda_{t+1}^u + \mu_{t+1}^u}{\pi_{t+1}} = \lambda_t^u \left[1 + \phi_M (M_t - M)\right] \tag{86}$$

$$\lambda_t^u \left(1 + \varkappa^{DC} \frac{DC_t^u}{\widetilde{DC}^u}\right) = \beta \mathbb{E}_t \xi_{t+1} \lambda_{t+1}^u \frac{R_t^{DC}}{\pi_{t+1}} \tag{87}$$

$$\alpha_M C_t^u = \frac{M_{t-1}}{\pi_t} \tag{88}$$

Production.

$$Q_t = 1 + \frac{\kappa_I}{2} \left(\frac{I_t}{I} - 1\right)^2 - \frac{I_t}{I} \kappa_I \left(\frac{I_t}{I} - 1\right) \tag{89}$$

$$K_t = (1-\delta) K_{t-1} + I_t \tag{90}$$

$$Y_t = A_t K_{t-1}^\alpha L_t^{1-\alpha} \tag{91}$$

$$\frac{z_t^k K_{t-1}}{w_t L_t} = \frac{\alpha}{1-\alpha} \tag{92}$$

$$MC_t = \frac{1}{A_t} \left(\frac{z_t^k}{\alpha}\right)^\alpha \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{93}$$

$$\pi_t(\pi_t - 1) = \frac{\epsilon - 1}{\kappa} \left(\mathcal{M}_t MC_t + \tau - 1\right) + \mathbb{E}_t \Lambda_{t,t+1}^h (\pi_{t+1} - 1) \pi_{t+1} \frac{Y_{t+1}}{Y_t} \tag{94}$$

Banks.

$$\psi_t = \theta^b \phi_t \tag{95}$$

$$\phi_t = \frac{v_t}{\theta^b - \mu_t} \tag{96}$$

$$\mu_t = \mathbb{E}_t \Omega_{t,t+1} \left[\frac{z^k_{t+1} + (1-\delta) Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}} \right] \tag{97}$$

$$v_t = \mathbb{E}_t \Omega_{t,t+1} \frac{R_t}{\pi_{t+1}} \tag{98}$$

$$\Omega_{t,t+1} = \Lambda^h_{t,t+1}(1 - \sigma_b + \sigma_b \psi_{t+1}) \tag{99}$$

Monetary and fiscal policy.

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_Y} \right]^{1-\rho_R} \exp(\varepsilon^R_t) \tag{100}$$

$$\frac{R^{DC}_{t-1}}{\Pi_t} DC_{t-1} + \frac{M_{t-1}}{\Pi_t} = \tau Y_t + \Gamma_h T^h_t + \Gamma_u T^u_t + DC_t + M_t \tag{101}$$

$$R^{DC}_t = R_t \tag{102}$$

Market clearing.

$$C_t = \Gamma_h C^h_t + \Gamma_u C^u_t \tag{103}$$

$$L_t = \Gamma_h L^h_t + \Gamma_u L^u_t \tag{104}$$

$$DC_t = \Gamma_h DC^h_t + \Gamma_u DC^u_t \tag{105}$$

$$\omega_t = 1 - \frac{C^u_t}{C^h_t} \tag{106}$$

$$\begin{aligned} Y_t &= C_t + \left[1 + \Phi\left(\frac{I_t}{I}\right)\right] I_t + \frac{\kappa}{2}(\pi_t - 1)^2 Y_t \\ &\quad + \Gamma_h(\chi^h_t + \chi^{DC,h}_t) + \Gamma_u(\chi^M_t + \chi^{DC,u}_t) \end{aligned} \tag{107}$$

$$K_t = \Gamma_h(K^h_t + K^b_t) \tag{108}$$

$$\begin{aligned} N_t &= \sigma_b \left[(z^k_t + (1-\delta) Q_t) K^b_{t-1} - \frac{R_{t-1}}{\pi_t} D_{t-1} \right] \\ &\quad + \gamma_b (z^k_t + (1-\delta) Q_t) \frac{K_{t-1}}{\Gamma_h} \end{aligned} \tag{109}$$

$$Q_t K^b_t = \phi_t N_t \tag{110}$$

$$Q_t K^b_t = D_t + N_t \tag{111}$$

Exogenous processes.

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon^A_t \tag{112}$$

$$\mathcal{M}_t = (1 - \rho_M)\mathcal{M} + \rho_M \mathcal{M}_{t-1} + \varepsilon^M_t \tag{113}$$

$$\xi_t = \rho_b \xi_{t-1} + \varepsilon^\xi_t \tag{114}$$

A.2.6 Model Steady State

In the non-stochastic steady state, we have the following:

$$Q = 1,$$

$$\pi = 1,$$
$$R = \frac{1}{\beta},$$
$$R^{DC} = R.$$

We define the discounted spreads on equity as:

$$s = \beta[z^k + (1-\delta)] - 1, \tag{115}$$

which we consider to be endogenous.

From the BHH's FOC with respect to equity, (70), we have:

$$\begin{aligned} 1 &= \beta \left[\frac{z^k + (1-\delta)}{1 + \varkappa^h \Gamma_h \frac{K^h}{K}} \right] \\ 1 + \varkappa^h \Gamma_h \frac{K^h}{K} &= \beta \left[z + (1-\delta) \right] \\ \Gamma_h \frac{K^h}{K} &= \frac{s}{\varkappa^h}. \end{aligned} \tag{116}$$

Additionally, in steady state we have:

$$\begin{aligned} \Omega &= \beta(1 - \sigma_b + \sigma_b \psi), \\ v &= \frac{\Omega}{\beta}, \\ \mu &= \Omega \left[z^k + (1-\delta) - \frac{1}{\beta} \right], \end{aligned}$$

and so, using (115) we can write:

$$\frac{\mu}{v} = s.$$

Next, define J as:

$$J = \frac{n_{t+1}}{n_t} = \left[z^k + (1-\delta) \right] \frac{K^b}{N} - R \frac{D}{N},$$

and use the following:

$$\begin{aligned} \frac{D}{N} &= \phi - 1, \\ \phi &= \frac{K^b}{N}, \end{aligned}$$

to write J as:

$$\begin{aligned} J &= (z^k + (1-\delta) - R)\phi + R \\ &= \frac{1}{\beta} \left[s\phi + 1 \right]. \end{aligned}$$

Then, from (39) we have:

$$\begin{aligned}
N &= \sigma_b \left\{ \left[z^k + (1-\delta) \right] K^b - RD \right\} + \gamma_b \left[z^k + (1-\delta) \right] \frac{K}{\Gamma} \\
\frac{N}{N} &= \sigma_b \left\{ \left[z^k + (1-\delta) \right] \frac{K^b}{N} - R\frac{D}{N} \right\} + \frac{\gamma_b}{N} \left[z^k + (1-\delta) \right] \frac{K}{\Gamma} \\
\beta &= \sigma_b \beta J + \frac{\gamma_b}{N} \beta \left[z^k + (1-\delta) \right] \frac{K}{\Gamma} \\
&= \sigma_b \beta J + \frac{\gamma_b K^b}{N} \left(1 + \varkappa^h \Gamma \frac{K^h}{K} \right) \frac{K}{\Gamma K^b} \\
&= \sigma_b \beta J + \gamma_b (1+s) \phi \frac{1}{\frac{\Gamma K^b}{K}} \\
&= \sigma_b \beta J + \gamma_b (1+s) \phi \frac{1}{\frac{K - \Gamma K^h}{K}} \\
&= \sigma_b \left[s\phi + 1 \right] + \gamma_b (1+s) \phi \frac{1}{1 - \frac{s}{\varkappa^h}} \\
\beta &= \sigma_b + \left[\sigma_b s + \gamma_b \frac{1+s}{1 - \frac{s}{\varkappa^h}} \right] \phi,
\end{aligned}$$

or

$$\phi = \frac{\beta - \sigma_b}{\sigma_b s + \gamma_b \frac{1+s}{1 - \frac{s}{\varkappa^h}}}$$

(22) in steady state gives us:

$$\begin{aligned}
\psi &= \beta(1 - \sigma_b + \sigma_b \psi) J \\
&= \beta J - \beta \sigma_b J + \beta \sigma_b \psi J \\
&= \beta(1 - \sigma_b) J + \beta \sigma_b \psi J \\
&= \frac{\beta(1-\sigma_b) J}{1 - \beta \sigma_b J} \\
&= \frac{(1-\sigma_b) \left[s\phi + 1 \right]}{1 - \sigma_b \left[s\phi + 1 \right]} \\
&= \frac{(1-\sigma_b) \left[s\phi + 1 \right]}{1 - \sigma_b - \sigma_b s \phi},
\end{aligned}$$

and from (77) we have

$$\psi = \theta^b \phi.$$

Combine the expressions for ϕ and ψ to get:

$$\frac{\theta^b (\beta - \sigma_b)}{\sigma_b s + \gamma_b \frac{1+s}{1 - \frac{s}{\varkappa^h}}} = \frac{(1-\sigma_b) \left[\frac{s(\beta - \sigma_b)}{\sigma_b s + \gamma_b \frac{1+s}{1 - \frac{s}{\varkappa^h}}} + 1 \right]}{1 - \sigma_b - \sigma_b \left[\frac{s(\beta - \sigma_b)}{\sigma_b s + \gamma_b \frac{1+s}{1 - \frac{s}{\varkappa^h}}} \right]},$$

then rearrange:

$$H(s) = (1-\sigma_b)\left[s\beta + \gamma_b \frac{1+s}{1-\frac{s}{\varkappa^h}}\right]\left[s\sigma_b + \gamma_b \frac{1+s}{1-\frac{s}{\varkappa^h}}\right] - \theta^b(\beta-\sigma_b)\left[\sigma_b(1-\beta)s + (1-\sigma_b)\gamma_b \frac{1+s}{1-\frac{s}{\varkappa^h}}\right].$$

We can observe that as $\gamma_b \to 0$,

$$H(s) = (1-\sigma_b)s^2\beta\sigma_b - \theta^b(\beta-\sigma_b)\left[\sigma_b(1-\beta)s\right]$$
$$\implies s \to \theta^b \frac{(\beta-\sigma_b)(1-\beta)}{(1-\sigma_b)\beta}.$$

Thus, there exists a unique steady state equilibrium with positive spread $s > 0$ for a small enough γ_b.

Given s, we then yield:

$$z^k = \frac{1}{\beta}(1+s) - (1-\delta),$$

and from (9) in the steady state:

$$MC = \frac{1-\tau}{\mathcal{M}},$$

and with (92), (8), and (10) we get:

$$MC = \frac{z^k}{\alpha}\frac{K}{Y},$$

or

$$\frac{K}{Y} = MC\frac{\alpha}{z^k}.$$

From the FOCs of the BHH and UHH problem, we have:

$$w = \zeta_0^h (L^h)^\zeta,$$
$$w = \frac{\zeta_0^u (L^u)^\zeta (1 + \frac{\alpha_M}{\beta} - \alpha_M)}{\left[C^u - \zeta_0^u \frac{(L^u)^{1+\zeta}}{1+\zeta}\right]}.$$

But since we have that $\zeta_0^u = \frac{\zeta_0^h}{(1+\frac{\alpha_M}{\beta}-\alpha_M)}$, we can write:

$$w = \zeta_0^h L^\zeta.$$

We can then use our previous expression for w to express L as a function of z^k:

$$L = \left[\frac{1-\alpha}{\zeta_0^h}\left(\frac{z^k}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{1}{\zeta}}.$$

Since we know that

$$w = (1-\alpha)\frac{Y}{L},$$

we yield:

$$Y = \frac{\zeta_0^h}{\alpha}\left[\frac{1-\alpha}{\zeta_0^h}\left(\frac{z^k}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{1+\zeta}{\zeta}}.$$

Additionally, we have:

$$\frac{I}{K} = \delta,$$

and

$$\frac{1}{\beta} = \frac{\alpha\frac{Y}{K} + 1 - \delta}{1 + \varkappa^h \Gamma_h \frac{K^h}{K}}$$

$$\Leftrightarrow \frac{Y}{K} = \frac{\beta^{-1}(1+s) + \delta - 1}{\alpha},$$

from (116), and

$$\frac{I}{Y} = \frac{I/K}{Y/K} = \frac{\alpha\delta}{\beta^{-1}(1+s) + \delta - 1}.$$

These of course imply:

$$K = \left[\frac{1-\alpha}{\zeta_0^h}\left(\frac{z^k}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{1+\zeta}{\zeta}} \frac{\zeta_0^h}{\beta^{-1}(1+s) + \delta - 1}$$

With K and s in hand, we can then turn back to the BHH's FOC wrt to equity, (70), to find K^h:

$$K^h = \frac{s}{\varkappa^h}\frac{K}{\Gamma_h},$$

and also get K^b:

$$K^b = \frac{K}{\Gamma_h} - K^h.$$

This then gives us N as we already solved ϕ:

$$N = \frac{K^b}{\phi}.$$

Then D is also solved as a residual from (18):

$$D = K^b - N.$$

Given Y, I, and K, we can get C:

$$\frac{C}{Y} = 1 - \frac{I}{Y} - \frac{\varkappa^h}{2}(\Gamma_h K^h)^2 \left(\frac{K}{Y}\right)^{-1}.$$

From the UHH's FOC with respect to M, we have:

$$\mu^u = \lambda^u \left(\frac{1}{\beta} - 1\right),$$

and the FOC with respect to consumption gives us an expression for the marginal utility from consumption:

$$\left(C^u - \zeta_0^u \frac{(L^u)^{1+\zeta}}{1+\zeta}\right)^{-1} = \lambda^u \left(1 + \frac{\alpha_M}{\beta} - \alpha_M\right).$$

Thus, we can express λ^u as a function of the marginal utility of consumption:

$$\frac{1}{\lambda^u} = \left(1 + \frac{\alpha_M}{\beta} - \alpha_M\right)\left(C^u - \zeta_0^u \frac{(L^u)^{1+\zeta}}{1+\zeta}\right),$$

noting that because of the values of ζ_0^h and ζ_0^u, we have:

$$L^u = \left(\frac{w}{\zeta_0^h}\right)^{\frac{1}{\zeta}}.$$

Finally, much like aggregate digital currency holdings, the BHH will not hold any digital currency holdings in steady state due to the presence of management costs. This means that in steady state:

$$DC^h = \frac{\beta R^{DC} - 1}{\varkappa^{DC}} + \widetilde{DC}^h$$

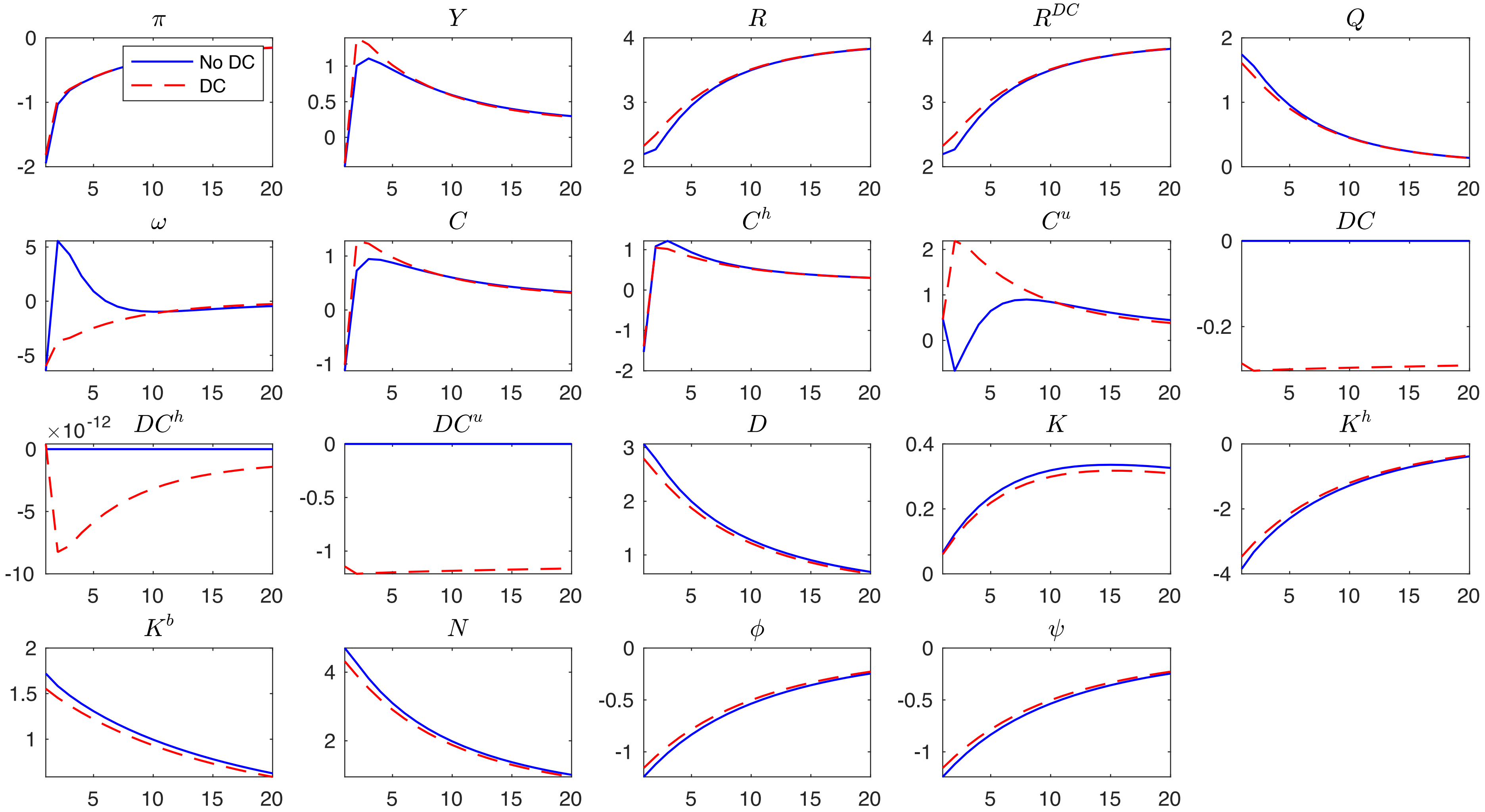
which, of course, implies:

$$DC^u = \frac{\beta R^{DC} - 1}{\varkappa^{DC}} + \widetilde{DC}^u.$$

A.2.7 Additional Impulse Responses to Shocks

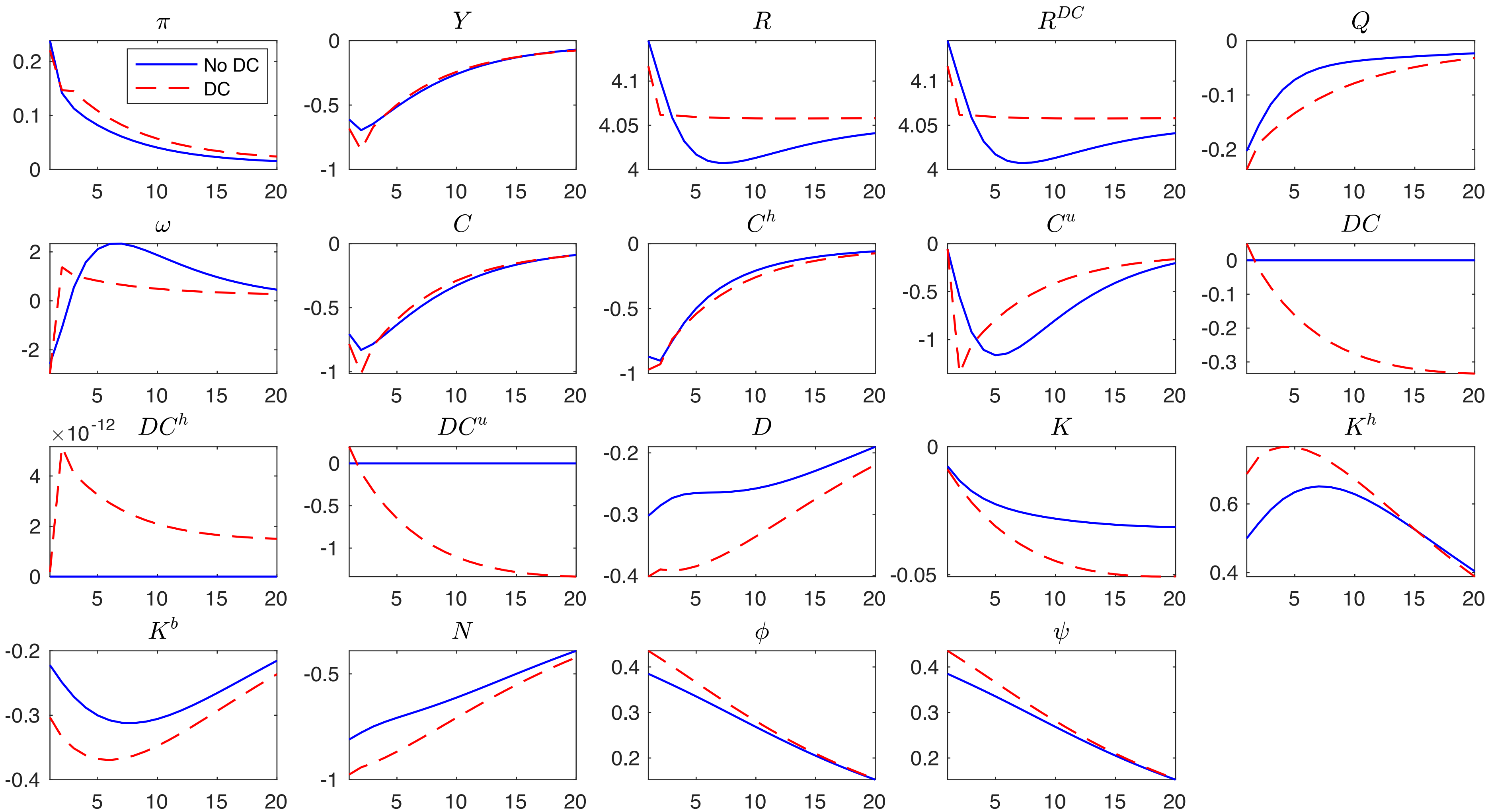
Figures 10, 11, and 12 present results in response to an annualised 1% orthogonal innovation to TFP, cost-push, and preference shocks, respectively. The figures compare IRFs for a no-CBDC economy and to a CBDC-equipped economy.

Figure 10: **IRFs to a 1% TFP shock**



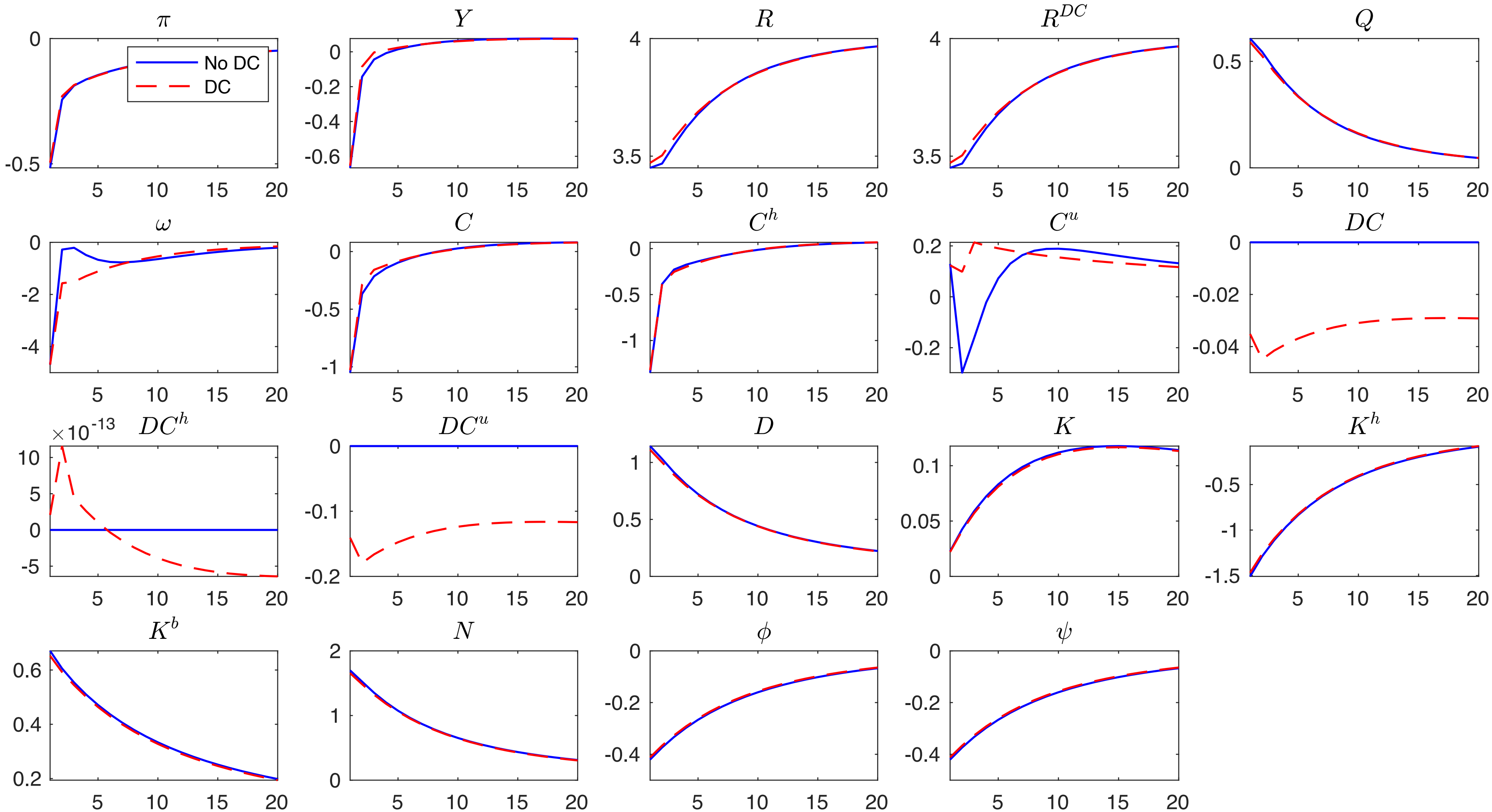
Note: Figure plots impulse responses of model variables with respect to a 1 % annualised innovation to TFP. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R), and Digital Currency Returns (R^{DC}) which are expressed as annualised net rates.

Figure 11: **IRFs to a 1% cost-push shock**



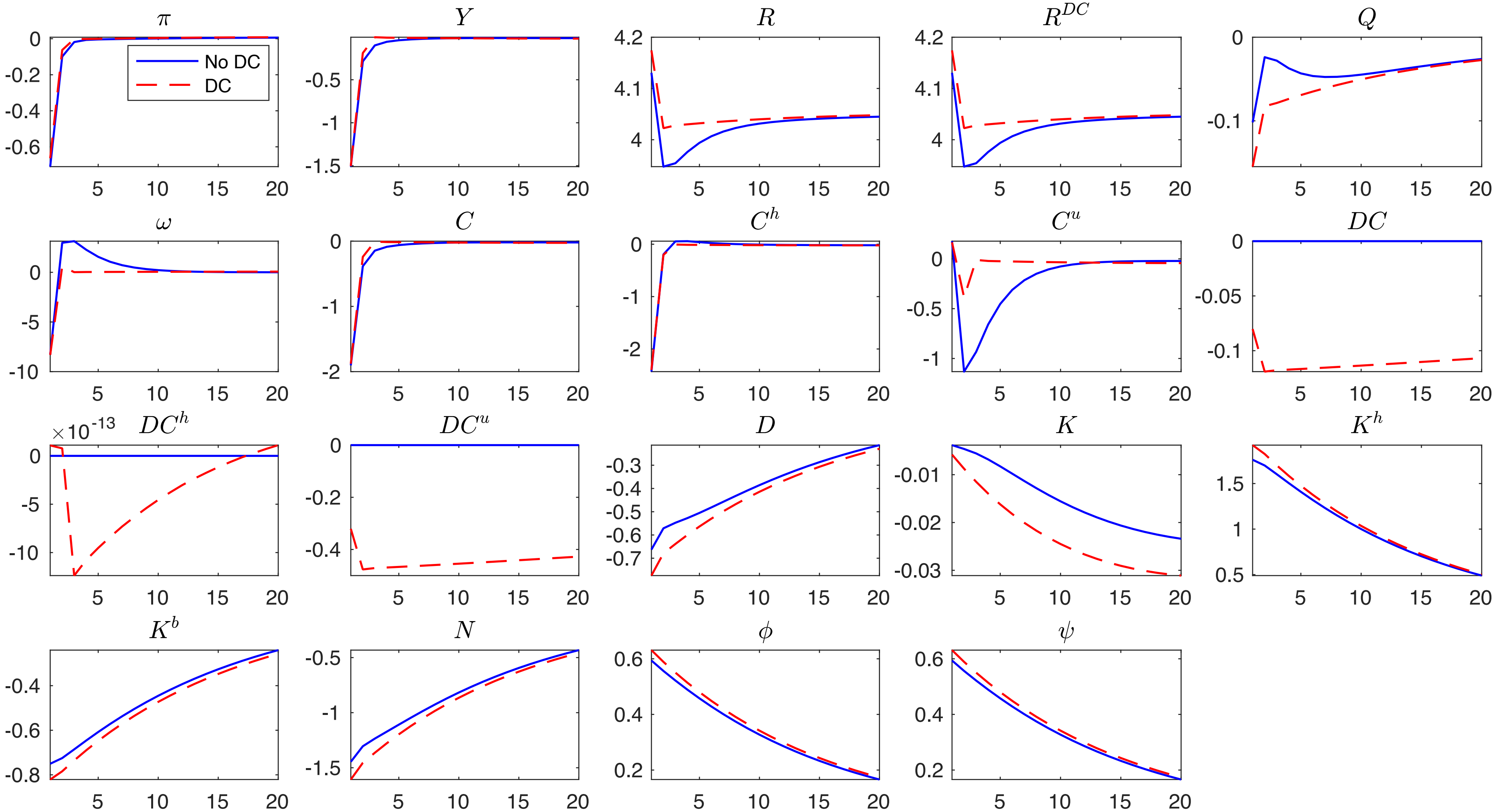
Note: Figure plots impulse responses of model variables with respect to a 1 % annualised innovation to markups. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R), and Digital Currency Returns (R^{DC}) which are expressed as annualised net rates.

Figure 12: **IRFs to a 1% demand shock**



Note: Figure plots impulse responses of model variables with respect to a 1% preference shock. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R) and Digital Currency Returns (R^{DC}) which are expressed as annualised net rates.

Figure 13: **IRFs to a 1% annualised monetary policy (tightening) shock**



Note: Figure plots impulse responses of model variables with respect to a 1% annualised monetary policy shock. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R) and Digital Currency Returns (R^{DC}) which are expressed as annualised net rates.