CBDCs, Financial Inclusion, and Optimal Monetary Policy

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Abstract

In this paper we study the macroeconomic effects of introducing a retail central bank digital currency (CBDC). Using a two agent framework and endowment economy with banked and unbanked households, we show CBDCs address financial inclusion of the unbanked by providing a savings vehicle to allow households to smooth consumption. Finally, we study the monetary policy implications in a New Keynesian setting. Welfare gains under Ramsey optimal monetary policy are higher for a retail CBDC with a primarily unbanked population. When CBDC and deposits are near substitutes, optimal policy requires the CBDC rate to track policy rates. Taken together, our findings suggest a stronger use case for CBDCs in emerging economies with a lower degree of financial inclusion.

Keywords: Central Bank Digital Currency, financial inclusion, inequality, monetary policy, optimal policy, Taylor rules, welfare

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1 Introduction

Central bank digital currencies (CBDC) are digital tokens, similar to a cryptocurrency, issued by a central bank. Central banks are actively studying the potential adoption of CBDCs, and notable examples include Sweden's E-Krona and China's Digital Currency Electronic Payment. In this emerging literature there is a focus on the macroeconomic effects (Kumhof et al. 2021; Benigno, Schilling, and Uhlig 2022; Ferrari Minesso, Mehl, and Stracca 2022), and implications for banking (Chiu et al. 2019; Skeie 2019; Keister and Sanches 2021; Agur, Ari, and Dell'Ariccia 2022). While CBDCs present obvious advantages – the increased financial inclusion of the unbanked population, improving cross-border payments, and facilitating fiscal transfers – there are still many unresolved issues in their design. For example, do CBDCs attenuate or amplify monetary policy transmission channels? Is the interest rate on the CBDC adjustable or fixed? Are there implications for redistribution through taxes and subsidies on CBDC? Is there a distinction between retail CBDCs which are distributed directly to households by the central bank or indirectly through the commercial banks?

In answering these questions, our paper focuses on CBDC design and in particular the financial inclusion effects of introducing a digital currency. The paper is divided into two parts. First, we review the arguments for and against a retail CBDC using a simple endowment economy with two types of agents. We then extend the model to examine the macroeconomic effects of issuing a digital currency when there is a banking sector and production, and a central bank that pursues output- and inflation-stabilising monetary policy. This framework allows us to evaluate monetary policy rules in a New Keynesian setup and determine the magnitude of monetary policy transmission for each CBDC design.

In the first part, we start with a simple two agent endowment economy with a representative banked household (BHH) and unbanked household (UHH). The unbanked use money while the banked have access to deposits.[1] We introduce a digital currency that can be used by the UHH as an alternative to cash. The central bank can pay an interest rate on this retail CBDC. The primary benefit of this digital currency is that it is a more effective savings vehicle as it relaxes the cash-in-advance (CIA) constraint of the UHH. Welfare for both sets of households improve with a retail CBDC.

In the next part of the paper, we then extend the model to include production

1. The BHH and UHH can be thought of as Ricardian and non-Ricardian households, respectively, as is typical in the two-agent New Keynesian literature. See, for example, Debortoli and Galí (2017).

and endogenous labour supply, monopolistic pricing of firms, a financial intermediary that lends to firms that use capital in production, and monetary policy set by a central bank. This setup allows us to evaluate optimal monetary policy rules and the role financial intermediaries play in the transmission effects of a retail CBDC. We conduct three tests using this model. First, we simulate the economy with respect to productivity shocks, cost-push inflationary shocks, and monetary policy shocks. Using these simulated responses, we address the question on whether a CBDC attenuates or amplifies the transmission of monetary policy. Our results suggest that monetary policy transmission is stronger upon introducing a retail CBDC, and amplifies the economy with respect to fundamental shocks to productivity. The intuition is straightforward: monetary policy has an additional lever in a CBDC economy as the central bank sets the rate on digital currency deposits, which tracks the policy rate in our specification. The UHH is therefore more sensitive to changes in the policy rate. This translates to more sensitive changes of bank net worth and leverage to monetary policy, amplifying the response of capital and production through bank balance sheets via a financial accelerator mechanism (Bernanke, Gertler, and Gilchrist 1999; Kiyotaki and Moore 1997, 2019).

Second, we use our model to evaluate the welfare effects of the introduction of the CBDC with respect to an economy with no digital currency. Similar to our endowment economy, we show distributional effects on welfare, with gains of CBDC adoption concentrated for the unbanked share of the population. In contrast, banked households benefit less from the introduction of a CBDC due to digital currency being an imperfect substitute for commercial bank deposits. A welfare analysis shows that the greatest use case for retail CBDCs therefore lies in an economy with low levels of financial inclusion. Third, we then evaluate optimal monetary policy to maximise welfare of households. The policy instruments include both the central bank rate on household deposits and the digital currency deposits rate. By conducting optimal monetary policy with two instruments, we can test alternative regimes for the CBDC monetary policy implementation. For example, should the CBDC rate be adjustable or fixed? The optimal policy results show that when CBDC deposits are a near substitute to regular deposits, it is optimal for the CBDC rate to track movements in the deposit rate. Welfare results show that a fixed rate in fact leads to net welfare losses in aggregate relative to the economy with no digital currency.

A final research question we answer is on elements of CBDC design and macroprudential policy. A first element draws on the distributive implications of introducing taxes on household digital currency deposits. A tax-neutral policy raises revenue from banked households and subsidises unbanked households. We show that these transfers increase aggregate welfare. The intuition is straightforward: the marginal welfare gains of subsidising CBDC deposits for the UHH exceeds the marginal welfare cost of taxing CBDC deposits for the BHH. A second element we test is the distributive implications of setting the CBDC rate above or below the policy rate. Consistent with our Ramsey optimal policy findings, the optimal rate setting for CBDC is the deposit rate. However, setting a CBDC rate at a constant spread above (below) the policy rate leads to net gains (losses) for the UHH, and net losses (gains) for the BHH. These distributional implications are apparent in the use case of CBDC for each type of household. While a higher CBDC rate benefits the unbanked, as they are offered a higher rate on savings, it can lead to negative effects on the banked through increasing the cost of capital and reducing the price of equity. A third element we test is whether the financial intermediaries should circulate retail CBDC deposits – which we call an indirect retail design – or, alternatively, be held at accounts with the central bank, which we call a direct retail design.[2] We find that across different settings of the CBDC rate, the direct retail design yields higher aggregate welfare. To explain our result, we note that the bank prefers to hold deposits relative to CBDC as CBDC incur adjustment costs. Therefore, BHH yield higher welfare in an equilibrium where they hold deposits directly with the central bank.

The remainder of the paper is structured as follows. In Section 1.1 we summarise the contributions of our paper to related literature. In Section 2 we outline the baseline endowment economy to clarify our intuition, and examine the welfare implications of introducing a CBDC. In Section 3 we introduce a two-agent New Keynesian (TANK) model with a banking sector. Using this model we examine welfare implications of introducing the CBDCs, including optimal policy exercises for when a social planner can set interest rates on both fiat and digital currencies. Section 4 considers CBDC design and macroprudential policy. Section 5 concludes the paper.

1.1 Related Literature

Our work relates to an emerging literature on the macroeconomic implications of CBDCs: Fernández-Villaverde et al. (2021), Andolfatto (2021), Benigno, Schilling, and Uhlig (2022), Chiu et al. (2019), Keister and Sanches (2021), Benigno (2019),

2. Some useful references are: https://voxeu.org/article/cbdc-architectures-financial-system-and-central-bank-future and https://voxeu.org/article/central-bank-digital-currencies-drivers-approaches-and-technologies.

George, Xie, and Alba (2020), Skeie (2019), Ikeda (2020), Kumhof et al. (2021), Cong and Mayer (2021), and Agur, Ari, and Dell'Ariccia (2022). The CBDC literature primarily focus on broad macroeconomic implications. For example, the domestic effects are documented in Kumhof et al. (2021). Skeie (2019) studies an equilibrium in which the cryptocurrency is susceptible to bank runs. The financial intermediation properties of CBDCs have been studied in Keister and Sanches (2021), which determines conditions in which the private sector is dis-intermediated with CBDC leading to welfare losses. Chiu et al. (2019) study the role of CBDCs when banks have market power, and show the introduction of CBDCs can lead to increased competition among banks, an increase in deposit rates and lending raising welfare. In contraste to these papers, our study focuses on the benefits of CBDCs in a two-agent framework. By studying households that do not have access to a financial asset, we focus on the financial inclusion benefits of a retail CBDC.

On the open economy front, Benigno, Schilling, and Uhlig (2022) model a two country framework in which a global stablecoin[3] is traded freely between both countries. They determine an equilibrium result of synchronisation of interest rates across the two countries in which users are indifferent between holding the global cryptocurrency and the domestic currency. Ferrari Minesso, Mehl, and Stracca (2022) setup a two country model with the CBDC issued by the home country. They find productivity spillovers are amplified in the presence of a CBDC, and it reduces the effectiveness of the foreign country's monetary policy. Cong and Mayer (2021) model the political economy of currency competition with countries choosing between adopting a CBDC and a private cryptocurrency. They show that emerging market economies with weak fundamentals can derive net welfare benefits from cryptocurrency adoption as an alternative to adopting a CBDC or the US Dollar. The novelty of our framework in this literature is to include an additional set of households (the unbanked) that do not have access to domestic banking channels. Critically, the unbanked only have access to digital currency as a medium of exchange and savings vehicle. Within this literature we are the first paper to evaluate the welfare benefits of the direct and indirect retail CBDC designs.[4]

The third part of the paper focuses on monetary policy transmission and interest rate rules. Ikeda (2020) models a two-country economy in which goods are priced in foreign currency. Domestic monetary policy transmission is weakened when prices

3. Such as Facebook's previously proposed Libra/Diem.

4. The taxonomy of direct and indirect retail CBDC designs is introduced in Auer and Böhme (2020). They provide many aspects of CBDC design, including architecture (whether it is a direct or indirect claim on the central bank), whether it uses a distributed ledger technology (DLT), account or token based or wholesale or retail. In this paper we focus solely on the architecture of CBDCs.

are denominated in a foreign currency. The channel of monetary policy transmission in Ikeda (2020) is expenditure switching; in our paper we offer an alternative channel through having digital currency deposits. Crucially, whether the system is retail or indirect retail matters for monetary policy transmission to bank balance sheets.

2 Two-Agent Endowment Economy Model with Central Bank Digital Currency

Below we introduce a simple two-agent endowment economy featuring CBDCs. The model comprises two types of households: the banked and the unbanked, denoted with $j = h$ and $j = u$, respectively. The population is normalised to unity, with the two types of households occupying the continuum $[0, 1]$. BHHs are proportion Γ_h of the population. They have access to a one-period risk-free savings asset, D_t, which pay a gross nominal rate of interest, R_t, and are in zero net supply. Conversely, the unbanked, of proportion $\Gamma_u = 1 - \Gamma_h$, do not have access to the risk-free savings asset.

However, our endowment economy features an additional asset, DC_t, which represents a CBDC or digital currency, and it is accessible and traded by both types of agents. In this simple setup, DC_t is in zero net supply, and, importantly, holdings of it earn a nominal return of R_t^{DC}.

The infinite horizon problem for the representative BHH is:

$$\mathbb{V}_t^h = \max_{\{C_{t+s}^h, D_{t+s}, DC_{t+s}^h\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u(C_{t+s}^h),$$

subject to the period budget constraint (in real terms):

$$C_t^h + D_t + DC_t^h + \chi_t^{DC} = T_t^h + \frac{R_{t-1}D_{t-1} + R_{t-1}^{DC} DC_{t-1}^h}{\pi_t},$$

where C_t^j, is consumption, T_t^j are lump sum transfers, and π_t is gross inflation,[5] and DC_t^j are digital currency balances held by households of type j. In our setup, $\chi_t^{DC,j}$ represents a cost of converting digital currencies for excess borrowing and lending,

5. Gross inflation, π_t, is defined as $\pi_t = P_t/P_{t-1}$, where P_t is the price level.

the extent of which is governed by cost parameter $\varkappa^{DC}$:[6]

$$\chi_t^{DC,j} = \frac{\varkappa^{DC}}{2}\left(DC_t^j\right)^2. \tag{1}$$

The analogous problem for the representative UHH is:

$$\mathbb{V}_t^u = \max_{\{C_{t+s}^u, DC_{t+s}^u\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u(C_{t+s}^u),$$

subject to their budget constraint,

$$C_t^u + M_t + DC_t^u + \chi_t^{DC,u} + \chi_t^M = T_t^u + \frac{M_{t-1} + R_{t-1}^{DC} DC_{t-1}^u}{\pi_t},$$

and the cash-in-advance (CIA) constraint,

$$\alpha_M C_t^u \leq \frac{M_{t-1}}{\pi_t}, \tag{2}$$

where χ_t^M are money adjustment costs of the form:

$$\chi_t^M = \frac{\phi_M}{2}(M_t - \bar{M})^2. \tag{3}$$

The CIA constraint features parameter $\alpha_M \in (0,1)$ which implies that even with digital currencies, the UHH must settle a certain fraction of their consumption purchasing decisions via real money holdings.

We also define ω_t as being an inequality measure, defined as:

$$\omega_t = 1 - \frac{C_t^u}{C_t^h}, \tag{4}$$

with higher (lower) values of ω_t showing an increase (decrease) in consumption inequality between the BHH and UHH in period t.

We assume that there exists a monetary authority which oversees real money balances. We assume the following law of motion for real money balances:

$$M_t = \frac{M_{t-1}}{\pi_t}. \tag{5}$$

Additionally, since DC_t is in zero net supply, we have the following aggregate con-

6. We note that as $\varkappa^{DC} \to 0$, the digital currency DC becomes a perfect substitute for the risk-free savings asset D; and so the quantities of digital currency are indeterminate in equilibrium.

dition:

$$DC_t^u = -\frac{\Gamma_h}{\Gamma_u} DC_t^h.$$

Finally, endowments are set exogenously and follow a stationary AR(1) process:

$$\ln T_t^i = \rho_T \ln T_{t-1}^i + \varepsilon_t^T,$$

where ε_t^T is an exogenous disturbance to both endowments with variance σ_T^2.

For a full set of equilibrium conditions for the economy economy both with and without CBDCs, please refer to Appendix A.1. We also show model impulse response functions (IRFs) to orthogonal shocks in Appendix A.1.5. Additionally, for a description of the parameterisation used in our analyses, please refer to Table 3 in Section 3.6.1.

2.1 Welfare Benefits of Financial Inclusion

To analyse the benefits of financial inclusion through the provision of CBDCs, we compare the ergodic mean[7] of welfare for the BHH, UHH, and a synthetic aggregate household:[8]

$$\mathbb{W}^j = F\left(\mathrm{Var}(\bar{C}^j), \mathrm{Var}(\bar{L}^j)\right), \quad j = \{h, u\}, \tag{6a}$$

$$\mathbb{W}^{agg} = \Gamma_h \mathbb{W}^h + \Gamma_u \mathbb{W}^u. \tag{6b}$$

2.1.1 CBDC Autarky

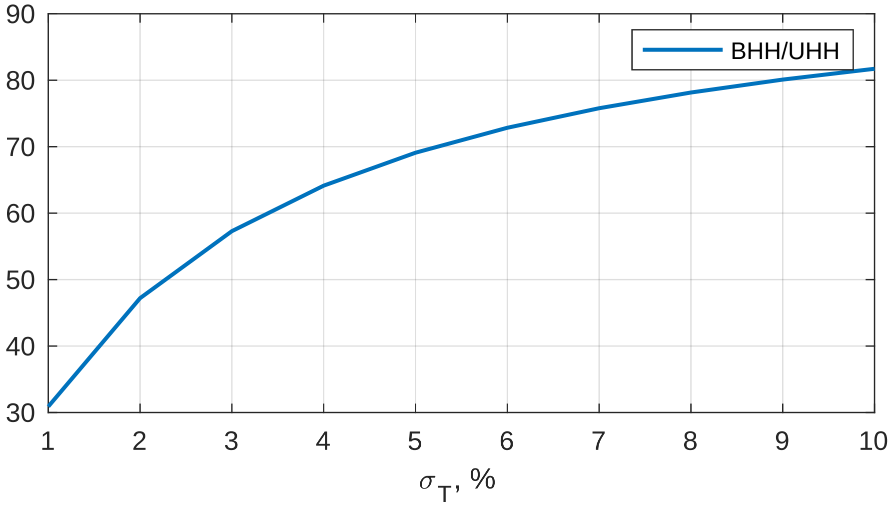
Figure 1 plots the ratio of welfare of the representative BHH to the UHH as a function of the variance of the endowment shock process in an economy with no CBDC. We show that as the variance of the endowment shock increases, the relative welfare of the banked increases. This supports our hypothesis of financial inclusion. As banked households have a savings vehicle to smooth consumption, their relative welfare increases when there are larger shocks to income. They are able to smooth consumption through the intertemporal consumption Euler equation. On the other hand, the unbanked can only smooth consumption through money holdings, and are impaired in their ability to smooth idiosyncratic shocks due to the presence of the CIA constraint. Based on our calibration, when the volatility of the endowment

7. To clarify, we take a second-order approximation about the deterministic steady state, subject the economy to our specified shocks, and then simulate the model for 2,000 periods to obtain the mean value of the variables of interest.

8. We adjust the population proportion of the representative BHH and UHH when constructing aggregate variables.

process is set to 1% (quarterly), the BHH sees an almost 30% welfare gain over the UHH; when volatility is set to 10% (quarterly), the BHH sees an over 80% welfare gain over the UHH.

Figure 1: Banked to unbanked relative welfare for a no-CBDC economy (% ch.)

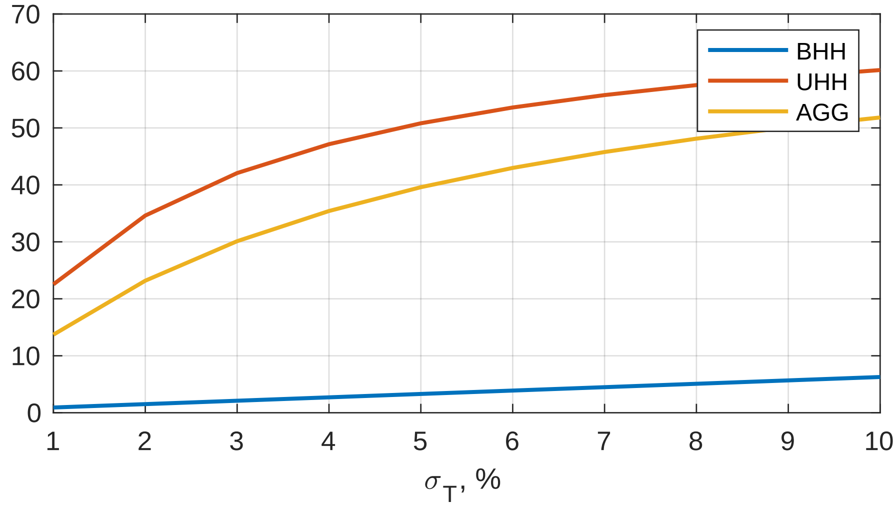


Note: Figure plots the ratio of the ergodic mean of welfare for the representative BHH and UHH, for increasing variance in the endowment shock process. The proportion of banked and unbanked households are set to a baseline value of $\Gamma_h = \Gamma_u = 0.5$.

2.1.2 CBDC Introduction

We highlight the importance of financial inclusion through the introduction of CBDC to both the BHH and UHH. In Figure 2 we plot the relative welfare gains for each representative household for the baseline endowment economy with CBDC over the endowment economy without CBDC. Our analysis shows that the welfare gains for the UHH are significant and scale with the idiosyncrasy of the endowment process.

Figure 2: Relative welfare and endowment volatility (% ch.)



Note: Figure plots the relative welfare gains of an economy with CBDC compared to an economy without CBDC for the representative BHH, UHH, and aggregate household for increasing variance in the endowment shock process. The proportion of banked and unbanked households are set to a baseline value of $\Gamma_h = \Gamma_u = 0.5$.

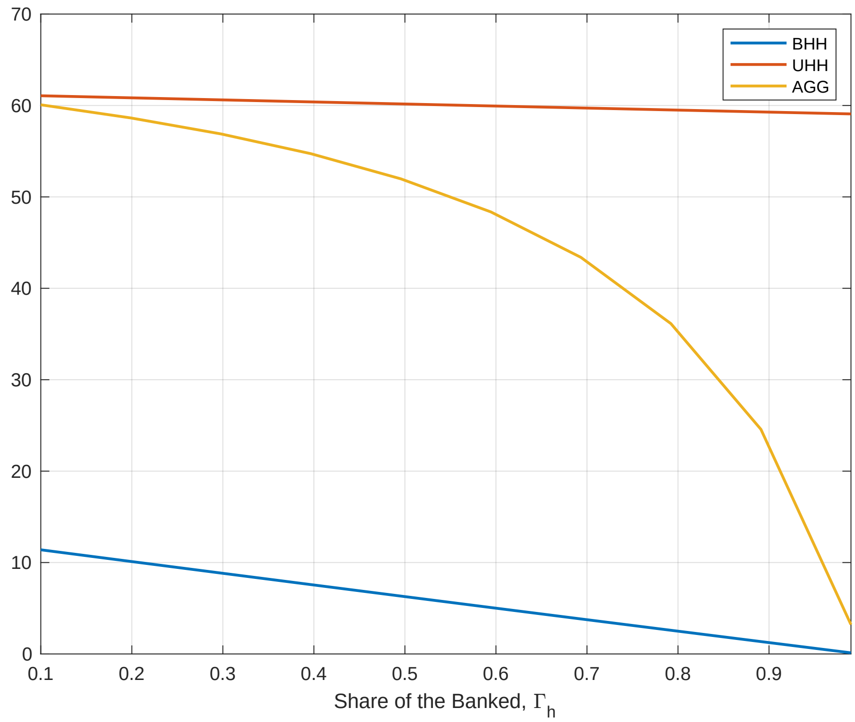
The results for two specific values of σ_T are highlighted in Table 1. With an endowment volatility of 1% (quarterly), the gains to BHH are 0.19%, and the gains to UHH are 22.4%. When the endowment volatility increases to 10% (quarterly), the relative welfare gain for the UHH in a CBDC-equipped economy over a no-CBDC economy is approximately 60%. In contrast, the welfare gains for the BHH are relatively low, primarily due to the fact that the BHH still have access to the first-best risk-free savings asset, D. Aggregate welfare increases to 51.2% above the economy with no CBDC.

Table 1: Relative welfare gains of a CBDC economy over a no-CBDC economy

	BHH	UHH	Agg.
$\sigma_T = 0.01$	0.19%	22.4%	14.1%
$\sigma_T = 0.1$	8.2%	60.4%	51.2%

Our analysis so far has focused on changing the volatility of the endowment shock, while keeping the shares of the banked and unbanked population fixed. In Figure 3, we plot the relative welfare gains for each representative household for the baseline endowment economy with CBDC over an endowment economy without CBDC, against the share of the banked population. In line with our previous results, aggregate welfare effects of the CBDC is decreasing in the share of BHH. When the share of BHH is 1, there are no gains from financial inclusion, and the relative welfare gains of introducing a CBDC are zero. In contrast, the relative gains of the CBDC economy approach 60% in aggregate when the share of BHH is 0.1. Taken together, our findings suggest that the welfare benefits of CBDCs are higher in economies with lower degrees of financial inclusion and a higher share of the unbanked.

Figure 3: Relative welfare and banked population (% ch.)



Note: Figure plots the relative welfare gains of an economy with CBDC compared to an economy without CBDC for the representative BHH, UHH, and aggregate household for increasing proportion of the banked population while keeping $\sigma_T = 0.1$ fixed.

3 Two-Agent New Keynesian Model with Central Bank Digital Currency

In this section, we extend the model presented in Section 2 in two ways: i) the introduction of a banking sector accompanied with credit frictions; and ii) a supply side of the economy with price stickiness and monopolistic competition (Christiano, Eichenbaum, and Evans 2005; Smets and Wouters 2007; Galí 2015) to build a two-agent New Keynesian (TANK) model as in Bilbiie (2018), Bilbiie and Ragot (2021), and Debortoli and Galí (2017, 2022).

We adopt the setup of Gertler and Karadi (2011), introducing a third type of agent – bankers – which allows us to maintain a representative setup of the household sector. In this setup, banked households hold claims on deposits – denominated in both fiat currency and digital currency – which are held at banks, and they may also directly invest in firms by purchasing equity holdings. Unbanked households are still limited to money holdings and digital currencies; the latter of which are also deposited into the banking sector. Banks then convert deposits into credit, facilitating loans to firms who acquire capital for the means of production, as in Gertler and Kiyotaki (2010, 2015).

3.1 Production

The supply side of the economy is simple. Final goods are produced by perfectly competitive firms that use labor and capital to produce their output. They also have access to bank loans, and conditional on being able to take out loan, they do not face any financial frictions. These firms pay back the crediting banks in full via profits. Meanwhile, capital goods are produced by perfectly competitive firms, which are owned by the collective household.

3.1.1 Final Good Firms

There is a representative competitive final good producing firm which aggregates a continuum of differentiated intermediate inputs according to a Dixit-Stiglitz aggregator:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} \, dj \right)^{\frac{\epsilon}{\epsilon-1}}, \epsilon > 0. \tag{7}$$

So final good firms maximize their profits by selecting how much of each intermediate good to purchase, and so their problem is:

$$\max_{Y_t(i)} P_t Y_t - \int_0^1 P_t Y_t(i) dj.$$

Solving for the FOC for a typical intermediate good j is:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t. \tag{8}$$

The relative demand for intermediate good j is dependent of j's relative price with ϵ, the price elasticity of demand, and is proportional to aggregate output, Y_t.

From Blanchard and Kiyotaki (1987), we can derive a price index for the aggregate economy:

$$P_t Y_t \equiv \int_0^1 P_t(i) Y_t(i) dj.$$

Then, plugging in the demand for good j from (8) we have:

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}.$$

3.1.2 Capital Good Firms

We assume that capital goods are produced by perfectly competitive firms, and that the aggregate capital stock grows according to the following law of motion:

$$K_t = I_t + (1-\delta)K_{t-1}, \tag{9}$$

where I_t is investment and $\delta \in (0,1)$ is the depreciation rate.

The objective of the capital good producing firm is to choose I_t to maximize revenue, $Q_t I_t$. Thus, the representative capital good producing firm's objective function is:

$$\max_{I_t} Q_t I_t - I_t - \Phi\left(\frac{I_t}{\bar{I}}\right) I_t,$$

where $\Phi(\cdot)$ are investment adjustment costs as in Christiano, Eichenbaum, and Evans (2005), and are defined as:

$$\Phi\left(\frac{I_t}{\bar{I}}\right) = \frac{\kappa_I}{2}\left(\frac{I_t}{\bar{I}} - 1\right)^2,$$

with $\Phi(1) = \Phi'(1) = 0$ and $\Phi''(\cdot) > 0$. The investment adjustment cost parameter, $\kappa_I = \Phi''(1)$ is chosen so that the price elasticity of investment is consistent with instrumental variable estimates in Eberly (1997).

Differentiating the objective function with respect to I_t gives the FOC:

$$Q_t = 1 + \Phi\left(\frac{I_t}{\bar{I}}\right) + \left(\frac{I_t}{\bar{I}}\right)\Phi'\left(\frac{I_t}{\bar{I}}\right). \tag{10}$$

3.1.3 Intermediate Goods Producers

The continuum of intermediate good producers are normalized to have a mass of unity. A typical intermediate firm i produces output according to a CRTS technology in capital and labor with a common productivity shock:

$$Y_t(i) = A_t K_{t-1}(i)^{\alpha} L_t(i)^{1-\alpha}.$$

The problem for the i-th firm is to minimize costs,

$$\min_{K_{t-1}(i),L_t(i)} z_t^k K_{t-1}(i) + w_t L_t(i),$$

subject to their production constraint:

$$A_t K_{t-1}(i)^\alpha L_t(i)^{1-\alpha} \geq Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t.$$

This yields the minimized unit cost of production:

$$MC_t = \frac{1}{A_t}\left(\frac{z_t^k}{\alpha}\right)^\alpha \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}. \tag{11}$$

The price-setting problem of firm i is set up à la Rotemberg (1982) where firm i maximizes the net present value of profits,

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty}\Lambda_{t,t+s}^h\left\{\left(\frac{P_{t+s}(i)}{P_{t+s}}(1-\tau) - MC_{t+s}\right)Y_{t+s}(i) - \frac{\kappa}{2}\left(\frac{P_{t+s}(i)}{P_{t-1+s}(i)} - 1\right)^2 Y_{t+s}\right\}\right],$$

by optimally choosing $P_t(i)$, and where κ denotes a price adjustment cost parameter for the firms.[9] Differentiating the above expression with respect to $P_t(i)$ yields the following FOC:

$$\begin{aligned}\kappa\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)\frac{Y_t}{P_{t-1}(i)} = {} & \frac{1-\tau}{P_t}\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t \\ & + \kappa\mathbb{E}_t\left[\Lambda_{t,t+1}^h\left(\frac{P_{t+1}(i)}{P_t(i)} - 1\right)\frac{P_{t+1}(i)}{P_t(i)^2}Y_{t+1}\right] \\ & - \epsilon\left(\frac{P_t(i)}{P_t}(1-\tau) - MC_t\right)\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon-1}\frac{Y_t}{P_t}.\end{aligned}$$

Evaluating at the symmetric equilibrium where intermediate firms optimally price

9. We calibrate κ to the following:

$$\kappa = \frac{\epsilon\theta}{(1-\theta)(1-\beta\theta)},$$

where θ is the probability of firm i being unable to optimally adjust its price in any given period as in a model with Calvo (1983) pricing. For further details please refer to Appendix A.2.1.

their output at $P_t(i) = P_t, \forall i$, allows us to write:

$$\begin{aligned} \pi_t(\pi_t - 1) &= \frac{1}{\kappa}\left[\epsilon MC_t + 1 - \epsilon + \tau\epsilon - \tau\right] \\ &\quad + \mathbb{E}_t\left[\Lambda^h_{t,t+1}(\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] + \xi^\pi_t, \end{aligned} \tag{12}$$

where ξ^π_t is a cost-push shock that follows a stationary AR(1) process (in logarithms).

Also, under the symmetric equilibrium we can express output as:

$$Y_t = A_t K^\alpha_{t-1} L^{1-\alpha}_t, \tag{13}$$

where it follows that:

$$K_{t-1} = \int_0^1 K_{t-1}(i)di, \quad L_t = \int_0^1 L_t(i)di.$$

As noted above, there is a distortion arising from monopolistic competition among intermediate firms. We assume that there is a lump-sum subsidy to offset this distortion, τ. From Equation (12), we see that the policy maker chooses a subsidy such that the markup over marginal cost is offset:[10]

$$\tau = -\frac{1}{\epsilon - 1}$$

which guarantees a non-distorted steady-state. Hereinafter, we abstract from distorted steady states and only consider the efficient steady state. Our choice to model nominal rigidity following Rotemberg pricing should not alter our welfare analysis in Section 3.6. As noted by Nisticò (2007) and Ascari and Rossi (2012), up to a second order approximation and provided that the steady state is efficient, models under both Calvo and Rotemberg pricing imply the same welfare costs of inflation. Therefore, a welfare-maximizing social planner would prescribe the same optimal policy across the two regimes.

3.2 Households and Workers

The representative household now contains a continuum of individuals, normalized to 1, each of which are of type $i \in \{b, h, u\}$. The setup follows Murakami and Viswanath-Natraj (2021). Bankers ($i = b$) and BHH workers share a perfect insurance scheme, such that they each consume the same amount of real output. However, UHH workers are not part of this insurance scheme, and so their consumption

10. Note that this assumes that steady state inflation is net-zero, i.e., $\bar{\pi} = 1$.

volumes are different from bankers and workers. Similar to before in Section 2, we define Γ_h as the proportion of the BHH and bankers, and the UHH are of proportion $\Gamma_u = 1 - \Gamma_h$.

We endogenize labor supply decisions on the part of households, and so the BHH maximize the present value discounted sum of utility:[11]

$$\mathbb{V}_t^h = \max_{\{C_{t+s}^h, L_{t+s}^h, D_{t+s}, K_{t+s}^h, DC_{t+s}^h\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \ln \left(C_{t+s}^h - \zeta_0^h \frac{(L_{t+s}^h)^{1+\zeta}}{1+\zeta} \right), \tag{14}$$

subject to their period budget constraint:

$$\begin{aligned} C_t^h + D_t + Q_t K_t^h + \chi_t^h + DC_t^h + \chi_t^{DC,h} + T_t^h \\ = w_t L_t^h + \Pi_t + (z_t^k + (1-\delta)Q_t)K_{t-1}^h + \frac{R_{t-1}D_{t-1} + R_{t-1}^{DC} DC_{t-1}^h}{\pi_t}, \end{aligned} \tag{15}$$

where w_t are real wages, $L_t^j, j \in \{h, u\}$, is labor supply, ζ is the inverse-Frisch elasticity of labor supply, ζ_0^j is a relative labor supply parameter, K_t^h are equity holdings in firms by the BHH, χ_t^h are the costs of equity acquisitions incurred by the BHH, T_t^j are now lump-sum taxes, Q_t is the price of equity/capital, and Π_t are distribution of profits due to the ownership of banks and firms. We also note that $\Lambda_{t,p}^h$ is the BHH stochastic discount factor (SDF):

$$\Lambda_{t,p}^h \equiv \beta^{p-t} \mathbb{E}_t \left(\frac{\lambda_p^h}{\lambda_t^h} \right), \tag{16}$$

where λ_t^h is the marginal utility of consumption for the BHH.

One distinction between the BHH and bankers purchasing equity in firms is the assumption that the BHH pays an efficiency cost when it adjusts its equity holdings. We assume the following functional form for χ_t^h:

$$\chi_t^h = \frac{\varkappa^h}{2} \left(\frac{K_t^h}{K_t} \right)^2 \Gamma_h K_t. \tag{17}$$

Meanwhile, the UHH maximizes the present discounted sum of per-period utilities given by:

$$\mathbb{V}_t^u = \max_{\{C_{t+s}^u, L_{t+s}^u, M_{t+s}, DC_{t+s}^u\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \ln \left(C_t^u - \zeta_0^u \frac{(L_t^u)^{1+\zeta}}{1+\zeta} \right), \tag{18}$$

11. We make use of Greenwood–Hercowitz–Huffman preferences for both the BHH and UHH to eliminate the income effect on an agent's labor supply decision. Additionally, it allows us to develop a tractable analytical solution for the model steady state

subject to its budget constraint,

$$C_t^u + M_t + \chi_t^M + DC_t^u + \chi_t^{DC,u} + T_t^u = w_t L_t^u + \frac{M_{t-1} + R_{t-1}^{DC} DC_{t-1}^u}{\pi_t}, \tag{19}$$

and the CIA constraint, (2).

3.3 Bankers and the Finance Sector

Among the population of bankers, each j-th banker owns and operates her own bank. The bankers are indexed on a continuum of measure one. A banker will facilitate financial services between households and firms by providing loans to firms in the form of equity, k_t^b, funded by domestic deposits, d_t, and digital currencies deposits, dc_t, and her own net worth, n_t. However, financial frictions may limit the ability of the banker to raise deposits from households.

To this end, each banker seeks to accumulate retained earnings to funds their investments. To maintain model tractability, in each period, bankers have a fixed probability of moving in and out of the financial sector. Let σ_b denote the probability that a banker remains as a banker in the following period, with complementary probability $1 - \sigma_b$ that she retires. This implies an expected franchise life of an individual bank of $\frac{1}{1-\sigma_b}$. Furthermore, the number of bankers exiting the financial market is matched by the number of new bankers entering.

New bankers start up their franchise with fraction γ_b of total assets of the banked households. Upon retirement, a banker will exit with her net worth, bringing the balance back to the household in the form of a dividend. Therefore, a banker will seek to maximize her franchise value, $\mathbb{V}_t^b$, which is the expected present discount value of future dividends:

$$\mathbb{V}_t^b = \mathbb{E}_t \left[\sum_{s=1}^{\infty} \Lambda_{t,t+s}^h \sigma_b^{s-1} (1 - \sigma_b) n_{t+s} \right], \tag{20}$$

where n_{t+s} is the net worth of the bank when the banker retires at date $t + s$ with probability $\sigma_b^{s-1}(1 - \sigma_b)$. Note that we make the simplifying assumption that each individual banker exogenoubsly accepts digital currency deposits, dc_t, directly in proportion to the population of bankers and total digital currency holdings. In other words, in aggregate, the total sum of individual digital currency deposits at each j-th bank, $dc_t(j)$, is equal to aggregate digital currency deposits, DC_t:

$$\int_0^1 dc_t(j)\, dj = DC_t.$$

Thus, a banker will choose quantities k_t^b and d_t to maximize expression (20). We assume that managing the sources of funding is costly in terms of resources, and so the banker pays the following management cost:

$$\chi_t^b = \frac{\varkappa^b}{2} x_t^2 Q_t k_t^b, \tag{21}$$

where we define $\varkappa^b > 0$ is a parameter and x_t is a banker's digital currency deposit leverage ratio:

$$x_t = \frac{dc_t}{Q_t k_t^b}. \tag{22}$$

A financial friction in line with Gertler and Kiyotaki (2010) is used to limit the banker's ability to raise funds, whereby the banker faces a moral hazard problem: the banker can either abscond with the funds she has raised from depositors, or the banker can operate honestly and pay out her obligations. Absconding is costly, however, and so the banker can only divert a fraction $\theta^b > 0$ of assets she has accumulated.

The caveat to absconding, in addition to only being able to take a fraction of assets away, is that it takes time – i.e. it take a full period for the banker to abscond. Thus, the banker must decide to abscond in period t, in addition to announcing what value of d_t she will choose, prior to realizing next period's rental rate of capital. If a banker chooses to abscond in period t, its creditors will force the bank to shutdown in period $t+1$, causing the banker's franchise value to become zero.

Therefore, the banker will choose to abscond in period t if and only if the return to absconding is greater than the franchise value of the bank at the end of period t, $\mathbb{V}_t^b$. It is assumed that the depositors act rationally, and that no rational depositor will supply funds to the bank if she clearly has an incentive to abscond. In other words, the bankers face the following incentive constraint:

$$\mathbb{V}_t^b \geq \theta^b Q_t k_t^b, \tag{23}$$

where we assume that the banker will not abscond in the case of the constraint holding with equality.

3.3.1 Bank Balance Sheet

Table 2 represents the balance sheet of a typical banker, and so we can write the following balance sheet constraint that the banker faces:

$$Q_t k_t^b + \chi_t^b = d_t + dc_t + n_t. \tag{24}$$

Table 2: Bank balance sheet

Assets	Liabilities + Equity
Loans $Q_t k_t^b$	Deposits d_t
Management costs χ_t^b	Digital currency deposits dc_t
	Net worth n_t

Additionally, we can write the flow of funds constraint for a banker as

$$n_t = [z_t^k + (1-\delta)Q_t]k_{t-1}^b - \frac{R_{t-1}}{\pi_t}d_{t-1} - \frac{R_{t-1}^{DC}}{\pi_t}dc_{t-1}, \tag{25}$$

noting that for the case of a new banker, the net worth is the startup fund given by the household:

$$n_t = \gamma_b[z_t^k + (1-\delta)Q_t]k_{t-1}.$$

3.3.2 Rewriting the Banker's Problem

With the constraints of the banker established, we can proceed to write the banker's problem as:

$$\max_{k_t, d_t} \mathbb{V}_t^b = \mathbb{E}_t \left[\Lambda_{t,t+1}^h \left\{(1-\sigma_b)n_{t+1} + \sigma_b \mathbb{V}_{t+1}^b\right\}\right],$$

subject to the incentive constraint (23) and the balance sheet constraint (24).

Since $\mathbb{V}_t^b$ is the franchise value of the bank, which we can interpret as a "market value", we can divide $\mathbb{V}_t^b$ by the bank's net worth to obtain a Tobin's Q ratio for the bank denoted by ψ_t:

$$\psi_t \equiv \frac{\mathbb{V}_t^b}{n_t} = \mathbb{E}_t \left[\Lambda_{t,t+1}^h(1-\sigma_b+\sigma_b\psi_{t+1})\frac{n_{t+1}}{n_t}\right]. \tag{26}$$

We define ϕ_t as the maximum feasible asset to net worth ratio, or, rather, the leverage ratio of a bank:

$$\phi_t = \frac{Q_t k_t^b}{n_t}. \tag{27}$$

Additionally, if we define $\Omega_{t,t+1}$ as the stochastic discount factor of the banker, μ_t as the excess return on capital over fiat currency deposits, μ_t^{DC} as the cost advantage of digital currency deposits over fiat currency deposits, and υ_t as the marginal cost of deposits, we can write the banker's problem as the following:

$$\psi_t = \max_{\phi_t} \left\{\mu_t\phi_t + \mu_t^{DC}x_t\phi_t + \left(1 - \frac{\varkappa^b}{2}x_t^2\phi_t\right)\upsilon_t\right\},$$

subject to

$$\psi_t \geq \theta^b \phi_t.$$

Solving this problem yields:

$$\psi_t = \theta^b \phi_t, \tag{28}$$

$$\phi_t = \frac{\upsilon_t}{\theta^b - \mu_t - \mu_t^{DC} x_t + \frac{\varkappa^b}{2} x_t^2 \upsilon_t}, \tag{29}$$

where:

$$\mu_t = \mathbb{E}_t \left[\Omega_{t,t+1} \left\{ \frac{z_{t+1}^k + (1-\delta) Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}} \right\} \right], \tag{30}$$

$$\mu_t^{DC} = \mathbb{E}_t \left[\Omega_{t,t+1} \left\{ \frac{R_t}{\pi_{t+1}} - \frac{R_t^{DC}}{\pi_{t+1}} \right\} \right], \tag{31}$$

$$\upsilon_t = \mathbb{E}_t \left[\Omega_{t,t+1} \frac{R_t}{\pi_{t+1}} \right], \tag{32}$$

$$\Omega_{t,t+1} = \Lambda_{t,t+1}^h (1 - \sigma_b + \sigma_b \psi_{t+1}). \tag{33}$$

For the complete solution of the banker, please refer to Appendix A.2.3 and A.2.4.

3.4 Fiscal and Monetary Policy

We assume that the government operates a simple fiscal rule to cover the producer subsidy addressing the distortions arising from monopolistic competition:

$$-\tau Y_t = \Gamma_h T_t^h + \Gamma_u T_t^u. \tag{34}$$

Meanwhile, the central bank is assumed to operate an inertial Taylor Rule for the nominal interest rate:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\rho_R} \left(\pi_t^{\phi_\pi} X_t^{\phi_Y} \right)^{1-\rho_R} \exp(\varepsilon_t^R) \tag{35}$$

where it reacts to inflation and the welfare relevant output gap, X_t, which we define as:

$$X_t = \frac{Y_t}{Y_t^f},$$

where Y_t^f is the flexible price level of output corresponding to when $\kappa = 0$, and where ε_t^R is an exogenous and transitory monetary policy shock.

Additionally, we assume that the central bank sets the nominal return on digital currency one-for-one in line with the nominal interest rate on deposits:

$$R_t^{DC} = R_t. \tag{36}$$

We explore the implications of alternative rules on model dynamics and welfare in Sections 3.6 and 3.7.

3.5 Market Equilibrium

Aggregate consumption, labor supply, and digital currency holdings by the BHH and UHH are given as:

$$C_t = \Gamma_h C_t^h + \Gamma_u C_t^u, \tag{37}$$

$$L_t = \Gamma_h L_t^h + \Gamma_u L_t^u, \tag{38}$$

$$DC_t = \Gamma_h DC_t^h + \Gamma_u DC_t^u. \tag{39}$$

The aggregate resource constraint of the economy is:

$$\begin{aligned} Y_t = C_t + \left[1 + \Phi\left(\frac{I_t}{\bar{I}}\right)\right] I_t + \frac{\kappa}{2}(\pi_t - 1)^2 Y_t \\ + \Gamma_h(\chi_t^h + \chi_t^b + \chi_t^{DC,h}) + \Gamma_u(\chi_t^M + \chi_t^{DC,u}), \end{aligned} \tag{40}$$

with aggregate capital being given by:

$$K_t = \Gamma_h(K_t^h + K_t^b). \tag{41}$$

Aggregate net worth of the bank is given by:

$$\begin{aligned} N_t = \sigma_b \left[(z_t^k + (1-\delta)Q_t)K_{t-1}^b - \frac{R_{t-1}}{\pi_t} D_{t-1} - \frac{R_{t-1}^{DC}}{\pi_t}\frac{DC_{t-1}}{\Gamma_h}\right] \\ + \gamma_b(z_t^k + (1-\delta)Q_t)\frac{K_{t-1}}{\Gamma_h}, \end{aligned} \tag{42}$$

and the aggregate balance sheet of the bank is given by the following equations:

$$Q_t K_t^b = \phi_t N_t, \tag{43}$$

$$\left(1 + \frac{\varkappa^b}{2} x_t^2\right) Q_t K_t^b = D_t + \frac{DC_t}{\Gamma_h} + N_t, \tag{44}$$

$$x_t = \frac{DC_t}{Q_t \Gamma_h K_t^b}. \tag{45}$$

Finally, the stationary AR(1) processes for TFP and cost-push shocks (in logs) are given by:

$$A_t = \rho_A A_{t-1} + \varepsilon_t^A, \tag{46}$$

$$\xi_t^\pi = \rho_\pi \xi_{t-1}^\pi + \varepsilon_t^\pi, \tag{47}$$

A competitive equilibrium is a set of seven prices, $\{\ MC_t,\ R_t,\ R_t^{DC},\ \pi_t,\ Q_t,\ w_t,\ z_t^k\ \}$, 19 quantity variables, $\{\ C_t,\ C_t^h,\ C_t^u,\ D_t,\ DC_t,\ DC_t^h,\ DC_t^u,\ I_t,\ K_t,\ K_t^b,\ K_t^h,\ L_t,\ L_t^h,\ L_t^u,\ M_t,\ N_t,\ T_t^h,\ T_t^u,\ Y_t\ \}$, six bank variables, $\{\ x_t,\ \psi_t,\ \phi_t,\ \mu_t,\ \mu_t^{DC},\ \upsilon_t\ \}$, and two exogenous variables, $\{\ A_t,\ \xi_t^\pi\ \}$. For a complete list of the equilibrium equations please refer to Appendix A.2.5.

3.6 Model Dynamics and Welfare Comparisons

3.6.1 Parameterisation and Steady State Values

Table 3: Parameter values

Parameter	Value	Description
θ	0.399	Elasticity of leverage wrt foreign borrowing
σ	0.940	Survival probability
γ^b	0.005	Fraction of total assets inherited by new banks
$\varkappa^b$	0.022	Management cost for DC
β	0.990	Discount rate
ζ	0.333	Inverse-Frisch elasticity
ζ_0^h	3.050	Labour supply capacity
$\varkappa^h$	0.020	Cost parameter of direct finance
Γ_h	0.500	Proportion of BHH
γ	0.500	CIA weight on money
ϕ_M	0.010	Money adjustment cost parameter
$\varkappa^{DC}$	0.0005	Digital currency adjustment cost parameter
α	0.333	Capital share of output
δ	0.025	Depreciation rate
ϵ	9.000	Elasticity of demand
κ_I	0.667	Investment adjustment cost
θ	0.750	Calvo parameter
τ	-0.125	Producer subsidy
$\mathcal{M}$	1.125	Markup

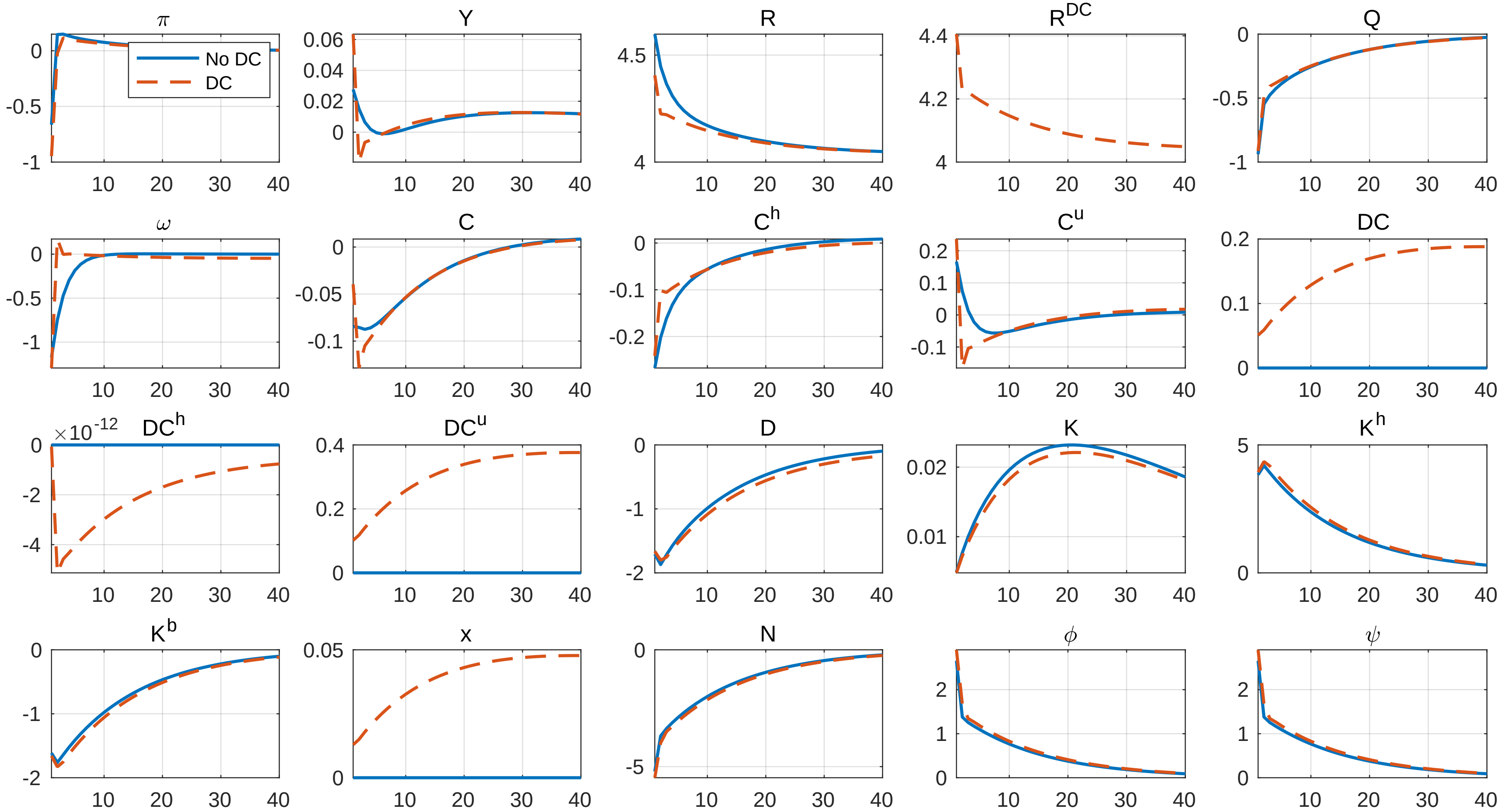
Table 3 – Continued

Parameter	Value	Description
ϕ_π	1.500	Taylor rule inflation coefficient
ϕ_Y	0.100	Taylor rule output coefficient
ρ_A	0.850	AR(1) coefficient for TFP shock
ρ_π	0.850	AR(1) coefficient for cost-push shock
ρ_R	0.550	Taylor rule persistence

Figure 4 presents results in response to a 1% annualised monetary policy shock. We assume a standard Taylor rule and allow the CBDC rate to track the deposit rate. Monetary policy is amplified when the CBDC is introduced. We see strong pass-through of policy rates to UHH consumption when they have access to CBDCs. The UHH response is quantitatively significant and translates to stronger transmission to aggregate consumption and the output gap. Turning to the bank balance sheet, we find neutral effects, where a contractionary shock induces the UHH to hold more CBDC, and the deposit base to shrink as the bank's funding costs increase. Therefore net worth of the bank and capital are quantitatively similar to the no CBDC regime. In summary, monetary policy transmission to aggregate consumption, output, and pass-through to inflation is strengthened with the introduction of the CBDC. For an analysis of fundamental-based shocks such as TFP and cost-push shocks refer to Appendix A.2.7.

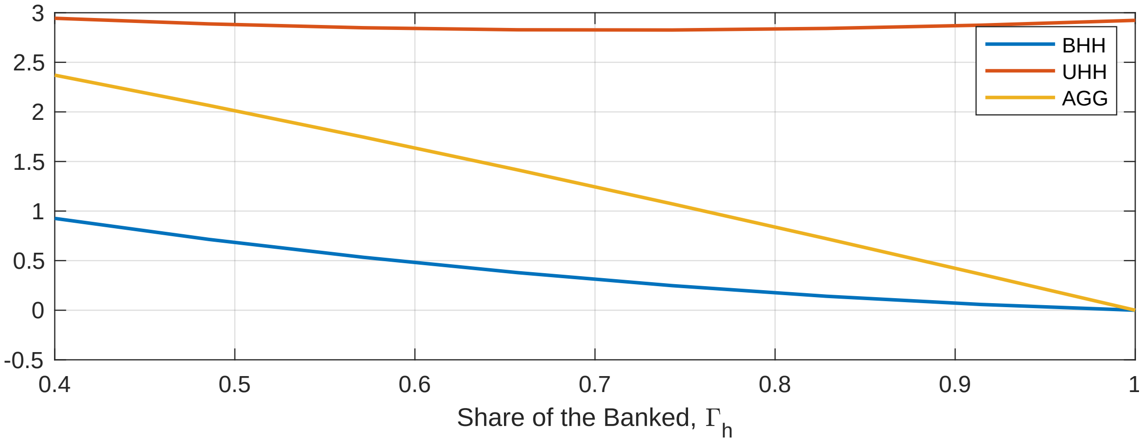
Figure 5 evaluates welfare of introducing a CBDC with respect to TFP, cost-push, and monetary shocks. We find the UHH have lower levels of financial inclusion and have a stronger incentive to adopt a retail CBDC. As the CBDC offers a rate of remuneration, it is an effective savings vehicle for the unbanked and enables them to achieve welfare gains through consumption smoothing. The BHH achieves smaller net welfare benefits with a retail CBDC. There are two reasons for this. First, the BHH face some distortionary costs of holding a CBDC relative to bank deposits, and therefore do not gain directly from access to a CBDC as they already have an efficient savings vehicle. However, when the UHH hold CBDC, this scales the bank balance sheet, increasing net worth and bank equity. This explains why BHH have higher welfare relative to the economy with no CBDC when the banked population share is low. Turning to aggregate welfare, we observe net welfare benefits relative to the no-CBDC regime when Γ_h is very low, so the economy is primarily unbanked. This corresponds to the case when the gains from financial inclusion are strongest.

Figure 4: IRFs to a 1% ann. monetary policy shock



Note: Figure plots impulse responses of model variables with respect to a 1% annualized innovation to the Nominal Interest Rate. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R) and Digital Currency Returns (R^{DC}) which are expressed as annualized net rates.

Figure 5: Welfare comparison (% change over no-CBDC regime)



Note: Figure plots welfare for BHH, UHH and aggregate households as a function for the share of the banked population, Γ_h. The welfare is calculated as a per cent change from the regime with no digital currency.

3.7 Optimal Policy

We now explore the implications for optimal policy, assuming that a policy maker has access to two instruments in order to maximise welfare: nominal interest rates on deposits, R, and nominal interest rates on digital currency, R^{DC}. More formally, let us state the problem for the welfare maximising policy maker as:

$$\max_{\{R_{t+s}, R^{DC}_{t+s}\}_{s=0}^{\infty}} \mathbb{V}_t = \Gamma_h \mathbb{V}_t^h + \Gamma_u \mathbb{V}_t^u, \tag{48}$$

subject to the entire set of structural equations as set out in Sections 3.1-3.5.

The purpose of this section is to investigate whether a policy that implies zero spread between the rates is optimal. We argue that R^{DC} is different to R in two distinct ways. First, digital currencies are – by construction – a sub-optimal savings instruments compared to deposits due to the presence of convex adjustment cost. The size and calibration of these adjustment costs have a significant impact on welfare outcomes.[12] Secondly, and as previously mentioned, we assume bankers cannot optimally select the quantity of digital currency deposits. Thus, R^{DC} can be used to induce a socially optimal level of digital currency to equity ratio, x, on the banker's balance sheet.

As CBDC and deposits are imperfect substitutes, the instruments available to the policy maker are not collinear, allowing us to conduct the optimal policy exercise. [13] The presence of Γ_u proportion of households that are unbanked in the economy,

12. We show these impacts and conduct a robustness check in Appendix A.2.8.
13. If bankers had been able to privately optimise x, and if digital currency deposits were a perfect

and which are are subject to a CIA constraint, leads the policy maker to set a steady-state rate of inflation that is deflationary – a result well covered in, for example, Chari, Christiano, and Kehoe (1991) and Schmitt-Grohé and Uribe (2010). Deflation is, however, costly through inefficient price adjustments; thus the policy maker induces a relatively low level of deflation (0.66% annualised). As $\Gamma_u \to 0$, the model converges to a standard representative agent setup and the optimal inflation level converges to zero, $\bar{\pi} \to 1$.

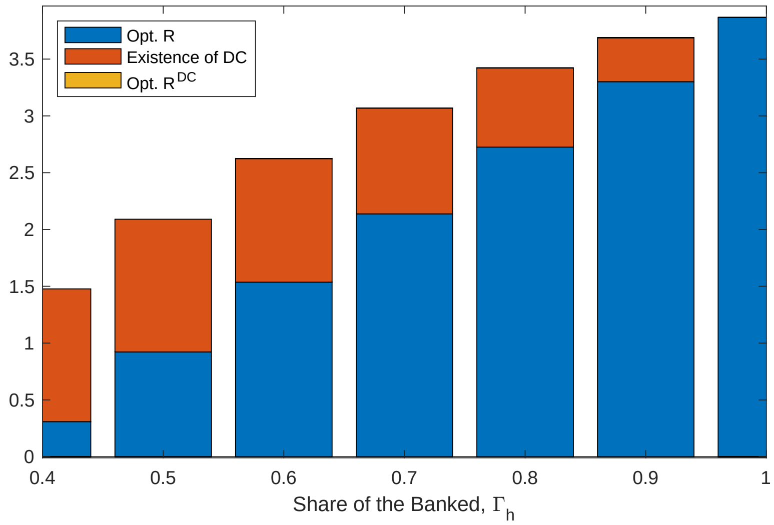
Figure 6 shows the decomposition of welfare gains associated with both the introduction of the CBDC and optimal monetary policy. For different levels of the banked population share, we decompose welfare improvements associated with the transition from the economy without digital currency and standard Taylor rule to the economy with digital currency and Ramsey-optimal two instrument monetary policy. These welfare gains are associated with: (i) introduction of digital currency, (ii) optimal conventional monetary policy, and (iii) optimal R_t^{DC} setting.

Firstly, we observe that for the economy with low initial financial inclusion ($\Gamma_h \to 0$) the welfare improvements are mainly associated with introduction of digital currency as a legal tender. As the proportion of UHH is relatively high, endowing them with digital currency leads to higher aggregate welfare. As their share decreases, we observe that the welfare benefits associated with provision of digital currency go to zero. Secondly, we see that as the proportion of BHH grows, the importance of optimal conventional monetary policy for welfare increases. Thirdly, our main finding is that deviating from $R_t = R_t^{DC}$ is welfare improving, but the welfare improvement is negligible and of order of numerical approximation error. The decomposition of the welfare gains with respect to the cost-push shock are shown in Figure 6a. Similar to our analysis with the TFP shock, we observe that the welfare improvement associated with introduction of digital currency diminish with the population share of BHH and that zero-spread policy is very close to optimal. The results are qualitatively similar with higher values of the adjustment costs of holding CBDC $\varkappa^{DC}$, and are available in the appendix A.2.8.

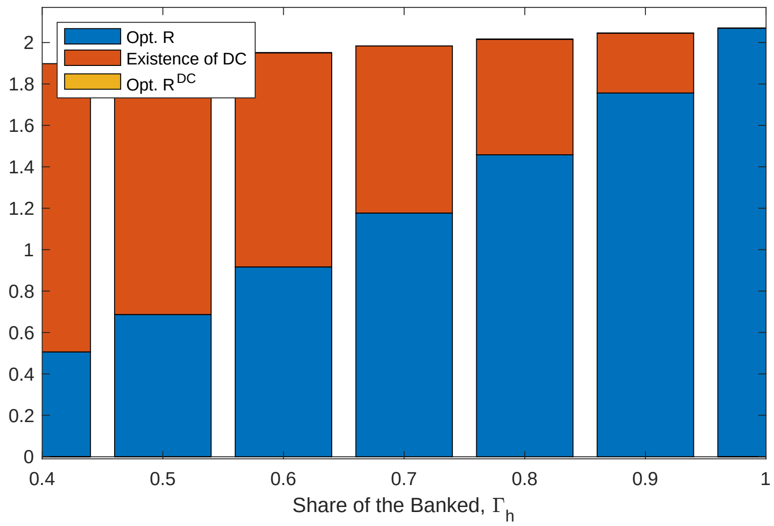
substitute for deposits, then the two instruments available to the policy maker would be collinear.

Figure 6: Welfare improvement decomposition

(a) TFP shock



(b) Cost-push shock



Note: Panel A: TFP shock, Panel B: cost-push shock. Vertical axis indicates % increase in welfare compared to baseline specification without digital currency access.

4 Macroprudential Policy and CBDC Design

4.1 Financial Taxes and Subsidies

We introduce macroprudential policy instruments in the form of taxes and subsidies to the bank balance sheet. Let τ_t^N denote the subsidy on bank net worth, τ_t^{DC} is a direct tax on digital currency holdings of the BHH and an equivalent subsidy to the UHH, and τ_t^K is a tax on bank equity holdings. The government's budget constraint would then be given by:

$$\tau_t^N \Gamma_h N_t - \tau Y_t = \tau_t^K \Gamma_h K_t^b + \Gamma_h (T_t^h + \tau_t^{DC,h} DC_t^h) + \Gamma_u (T_t^u + \tau_t^{DC,u} DC_t^u). \tag{49}$$

With these taxes and subsidies in place, we can rewrite the balance sheet constraint of an individual banker, (24), as:

$$\left(1 - \tau_t^K + \frac{\varkappa^b}{2} x_t^2\right) Q_t k_t^b = d_t + dc_t + (1 + \tau_t^N) n_t, \tag{50}$$

and the excess return on capital over fiat currency deposits, (30), and cost advantage of digital currency deposits over fiat currency deposits, (31), respectively, are defined as:

$$\mu_t = \mathbb{E}_t \left[\Omega_{t,t+1} \left\{ \frac{z_{t+1}^k + (1-\delta) Q_{t+1}}{Q_t} - (1 - \tau_t^K) \frac{R_t}{\pi_{t+1}} \right\} \right], \tag{51}$$

and the optimal leverage ratio of the banker, (29), is:

$$\phi_t = \frac{(1 + \tau_t^N) \upsilon_t}{\theta^b - \mu_t - \mu_t^{DC} x_t + \frac{\varkappa^b}{2} x_t^2 \upsilon_t}. \tag{52}$$

We first look at the effects of a permanent increase in tax and subsidy rates. Table 4 summarises the changes in of aggregate variables, as well as of welfare, in the deterministic steady state. We observe that, compared to baseline case, the subsidy to net worth, τ^N, and introduction of τ^{DC}, are welfare improving, while the tax on bank equity, τ^K, is not. The subsidy to net worth alleviates the inefficiencies associated with the competitive equilibrium in the presence of financial frictions. The subsidy to net worth increases the ability of the banker to finance equity and, thus, increases output and consumption. We note that even a small subsidy to net-worth of 0.1% induces increases in net worth by more than 2% and increase in output of more than 1%. The subsidy, however, increases volatility of the aggregate variables, most significantly that of deposits. Inflation volatility, however, declines, compared to baseline.

The tax and subsidy on digital currency, τ^{DC}, primarily serves the role of a redistribution device, since it is a tax on BHH CBDC holdings and a subsidy to UHH CBDC holdings. Given its design, it does not change the levels of output and consumption, but it does make deposits and net worth more volatile. This is due to the fact that the tax on BHH CBDC holdings makes the household more eager to substitute its digital currency holdings for deposits, which increases volatility thereof and, thus, makes bank net worth more volatile as well. The redistibutive properties of the instrument are manifested in an increase in the inequality measure, ω, that signifies a less egalitarian level of consumption across households. A striking increase in the aggregate welfare is a direct consequence of this redistribution and is mainly due to the concavity of the utility functions.

A tax on bank equity decreases the incentives of the banker to acquire equity and, thus, leads to a decline in aggregate equity and output. The introduction of this instrument, however, decreases volatility of the aggregate variables and leads to more equitable consumption levels across households. This is due to the fact that, by design, the UHH do not hold equity and benefit from the transfers from the BHH. The introduction of the tax, however, is not welfare improving in steady state due to a decrease in economic activity.

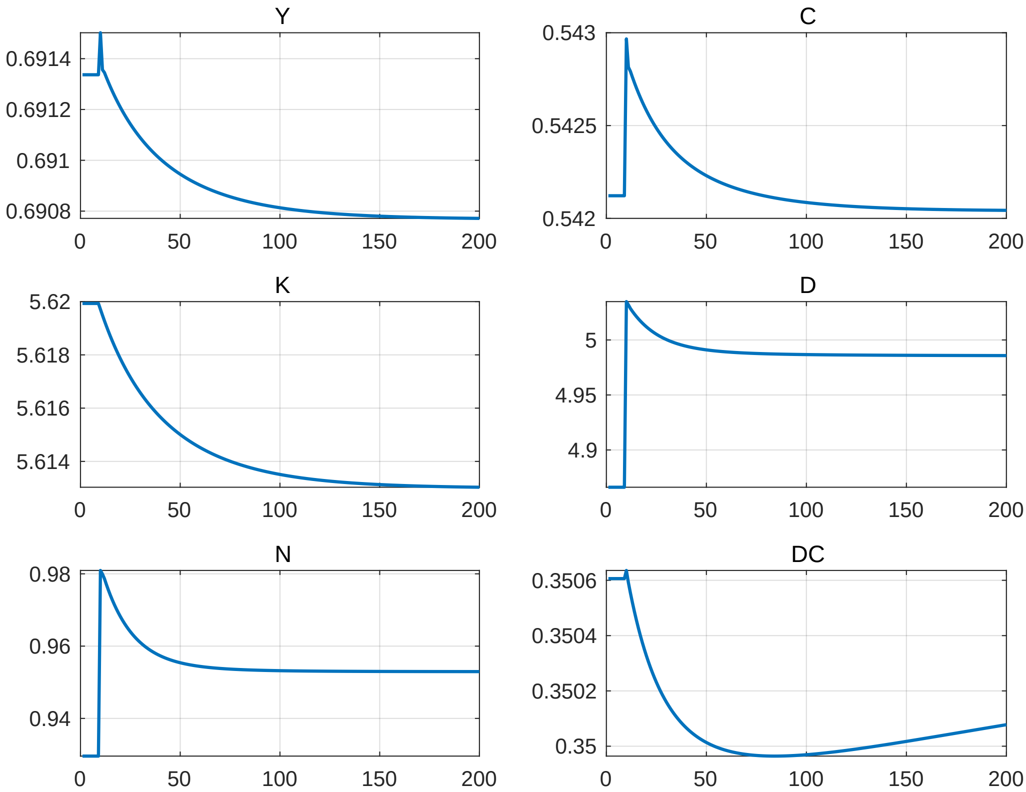
Table 4: Permanent tax policy changes

Variable	Baseline	τ^N only	τ^{DC} only	τ^K only
$\bar{Y}$	0.6898	0.6972	0.6898	0.6604
	(0.0077)	(0.0078)	(0.0077)	(0.0073)
$\bar{C}$	0.5447	0.5502	0.5437	0.5230
	(0.0092)	(0.0092)	(0.0092)	(0.0087)
$\bar{I}$	0.1401	0.1424	0.1401	0.1313
	(0.0021)	(0.0022)	(0.0021)	(0.0019)
$\bar{D}$	5.9744	6.1784	5.9745	5.1938
	(1.0204)	(1.0590)	(1.0852)	(0.9038)
$\bar{N}$	1.9048	1.9528	1.9048	1.7190
	(0.7242)	(0.7552)	(0.7229)	(0.6412)
$\bar{DC}$	0	0	0	0
	(0.0456)	(0.0494)	(0.1634)	(0.0387)
$\bar{Q}$	1	1	1	1
	(0.0722)	(0.0772)	(0.0722)	(0.0719)
$\bar{\pi}$	1	1	1	1
	(0.0107)	(0.0104)	(0.0107)	(0.0105)
$\bar{\omega}$	0.3806	0.3809	0.3257	0.3791
	(0.0087)	(0.0086)	(0.0141)	(0.0085)
$u(\bar{C}^h, \bar{L}^h)$	-111.5150	-110.5513	-118.8607	-115.4663
	(0.3197)	(0.3304)	(1.1893)	(0.3112)
$u(\bar{C}^u, \bar{L}^u)$	-260.9348	-260.5185	-235.5938	-262.8615
	(1.3410)	(1.4517)	(3.7737)	(1.1535)
$u(\bar{C}, \bar{L})$	-186.2249	-185.5349	-177.2272	-189.1639
	(0.5861)	(0.6322)	(1.3217)	(0.5087)

Note: Table shows deterministic steady state values and the standard deviation of variables simulated with TFP, cost-push, and monetary policy shocks over 2,000 periods.

The transition path to a new stochastic steady state after the introduction of permanent subsidy to net worth is shown in Figure 7. On impact, there is an increase in output and consumption, but over time their levels decrease and stabilise at new lower levels. Even though the policy induces higher deterministic steady state of the variables, it increases their volatility and, thus, their levels in stochastic steady state.

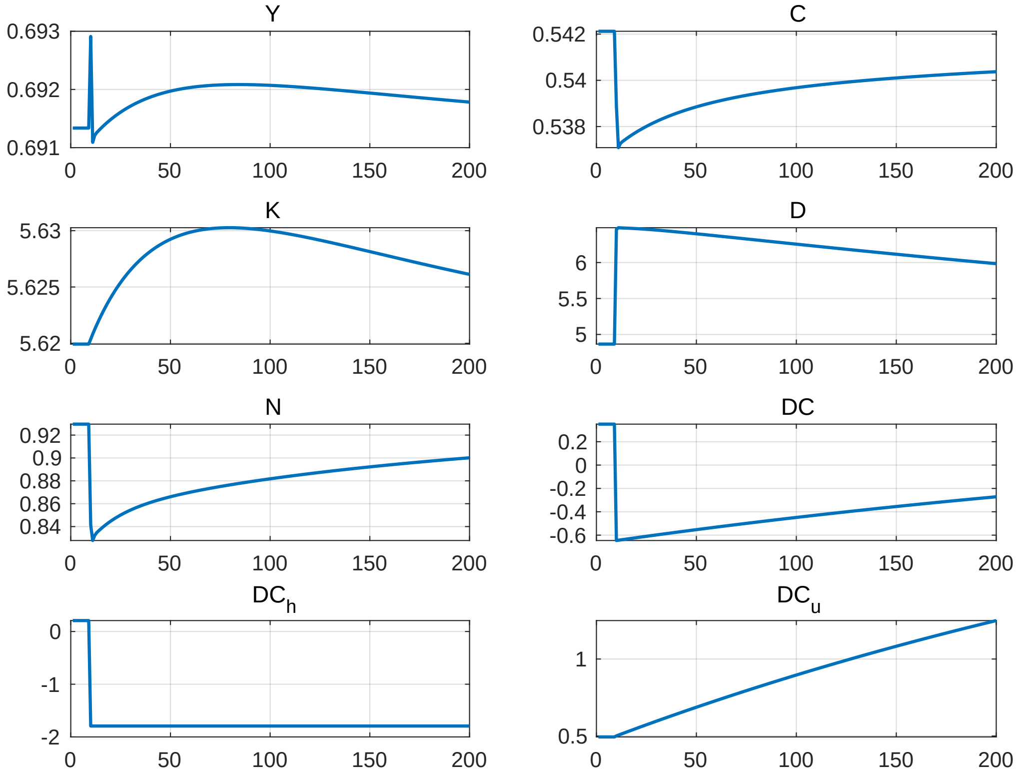
Figure 7: Stochastic steady state transition (permanent change to net worth subsidy, τ^N)



Note: Plots show a transition from the baseline stochastic steady to the new one induced by a permanent change in policy. The change in policy is assumed to happen in period 10 of the simulations.

The transition path to taxes and subsidies on CBDC is shown in Figure 8. The figure highlights the main redistibutional property of this permanent policy change; a tax on BHH CBDC holdings induces a sharp fall in DC^h, while a subsidy to UHH increases its CBDC holdings. The BHH switches to holding more deposits and equity. The banker's net worth, however, declines due to a decrease in the amount of digital currency it receives.

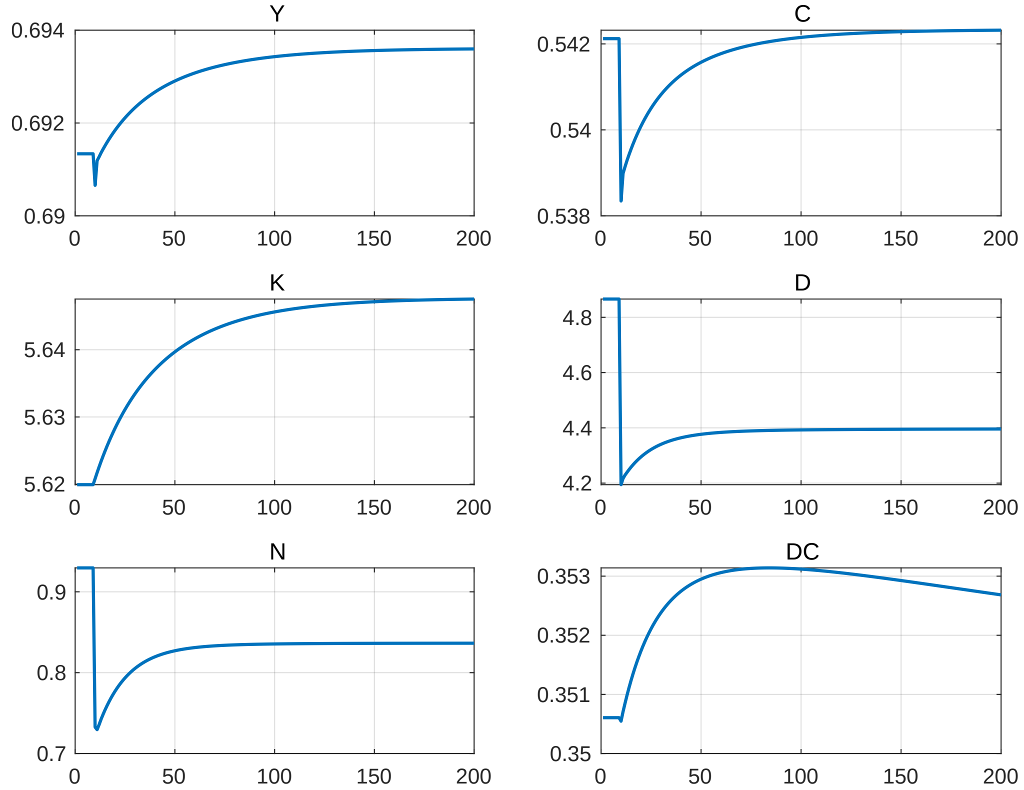
Figure 8: Stochastic steady state transition (permanent change to tax/subsidy on digital currency, τ^{DC})



Note: Plots show a transition from the baseline stochastic steady to the new one induced by a permanent change in policy. The change in policy is assumed to happen in period 10 of the simulations.

The transition path to a permanent tax on bank equity is shown in Figure 9. As the bank has a lower incentive to hold equity after the policy is introduced, it scales down its liabilities; deposits and net worth decline sharply on impact. The BHH, however, do not face this tax on their equity and, thus, decrease their deposit holdings and substitute them with digital currency and equity, which manifests in growth of aggregate equity.

Figure 9: Stochastic steady state transition (permanent change to tax on bank equity, τ^K)



Note: This Figure shows a transition from the baseline stochastic steady to the new one induced by a permanent change in policy. The change in policy is assumed to happen in period 10 of the simulations.

4.2 Constant Spread Rules and CBDC Design

Throughout this paper, we have made two important assumptions about digital currency implementation and related monetary policy design. Firstly, we assumed that $R^{DC} = R$ in the simple rule context. Secondly, commercial banks passively receive digital currency deposits and use them to purchase equity. In this subsection we relax these assumptions. We show that while deviating from $R^{DC} = R$ does not improve aggregate welfare, it induces different distributional outcomes, namely the UHH benefit from $R^{DC} > R$ and the BHH are better-off under $R^{DC} < R$. We also show implications of different digital currency designs on welfare.

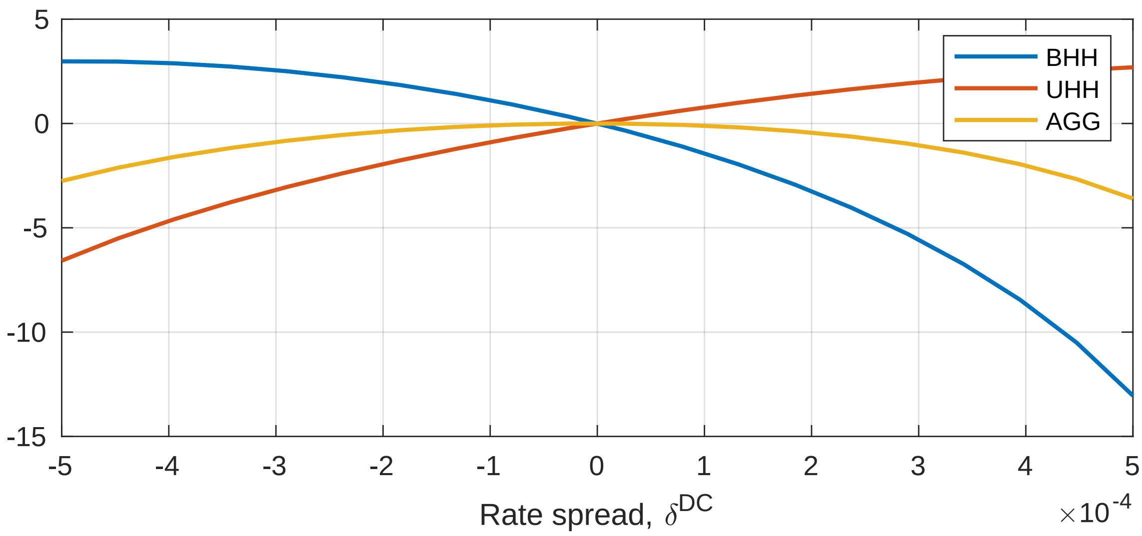
4.2.1 Welfare Implications of Constant Spread Rules

Assume that the policymaker operates the digital currency interest rate rule of the form,

$$R_t^{DC} = R_t + \delta^{DC},$$

where δ^{DC} is the constant spread term. We compare welfare outcomes of the individual households and the aggregate welfare measure to the baseline case, where $\delta^{DC} = 0$ and plot the corresponding welfare ratios with respect to a 1% TFP shock.

Figure 10: Welfare implications of constant spread rule



Note: Figure plots welfare for BHH, UHH and aggregate households as a function of the spread between the policy rate and the CBDC rate. The welfare is calculated as a per cent change from the regime with no digital currency.

We observe that while aggregate welfare is maximised with $R^{DC} = R$, positive and negative values of δ^{DC} imply different distributional outcomes. The BHH are better off under negative values of the spread, while the converse is true for UHH. The UHH benefit through the savings channel, where their digital currency deposits receive a higher rate of interest. The BHH are worse off because the banker is forced to hold digital currency deposits at a higher rate. We investigate if these effects can be mitigated though a different CBDC design, where digital currency does not appear on the balance sheet of the banker – commonly referred to as a 'retail direct' (RD) scheme, where digital currency is a direct claim on the central bank.

4.2.2 CBDC Design

As discussed, we assumed that all digital currency deposits appear on the banker's balance sheet. We depart from this assumption and assume that only a fraction of digital currency deposits, γ^{DC}, appear on the banker's balance sheet. Digital currency deposit claims on the [private] bank, thence, are denoted by:

$$DC_t^B = \gamma^{DC} DC_t.$$

Next, we first compare welfare outcomes of the households under different values of γ_{DC}. In Figure 11a, we observe that the RD regime ($\gamma^{DC} = 0$) is marginally welfare improving under any population composition[14] for the case when the spread between CBDC and deposit rates are zero. We then look at the two distinct cases with positive and negative constant spread δ^{DC}. We consider the case where $\delta^{DC} = 0.05\%$ in Figure 11b. We observe that the positive spread is marginally welfare improving under the RD regime on aggregate, and implies welfare benefits (losses) for the UHH (BHH). We also observe that there is a break-even point at $\gamma^{DC} = 0.5$. In Figure 11c, we proceed with the negative spread case, $\delta^{DC} = -0.05\%$. We observe that while the negative spread is not welfare improving on aggregate, the RD regime is more optimal. The result is robust to different population compositions. The findings are summarised in Table 5.

Table 5: Welfare implications of different digital currency designs

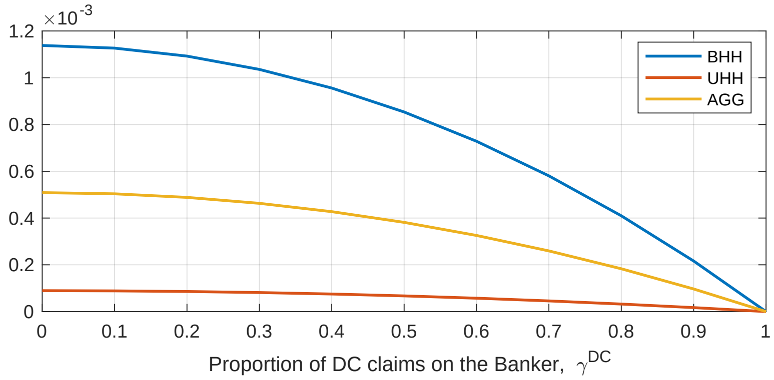
	$\delta^{DC} = 0.05\%$		$\delta^{DC} = -0.05\%$		$\delta^{DC} = 0\%$	
Regime	RD	RI	RD	RI	RD	RI
BHH	-3.53	-13.05	2.86	2.98	0.0011	0
UHH	3.87	2.69	-3.86	-6.58	0.0001	0
AGG	0.91	-3.6	-1.18	-2.76	0.0005	0

Note: Table summarises welfare for the BHH, UHH and aggregate households as a function of the share of the spread between the policy rate and the CBDC rate. The welfare is calculated as a percent change from the regime with no digital currency for the retail direct (RD) and retail indirect (RI) regimes.

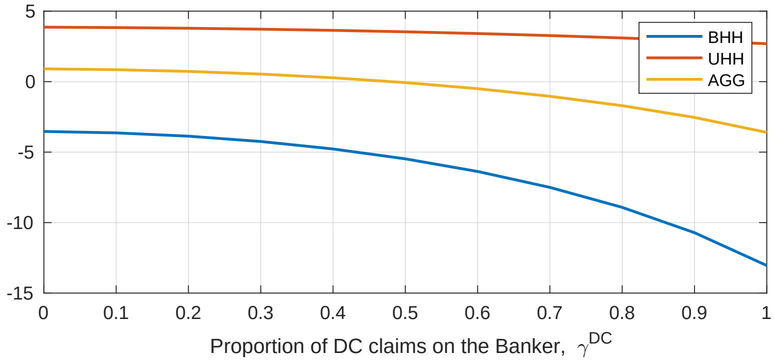
14. Refer to Appendix A.2.8 for the robustness checks.

Figure 11: Welfare implications of different CBDC regimes

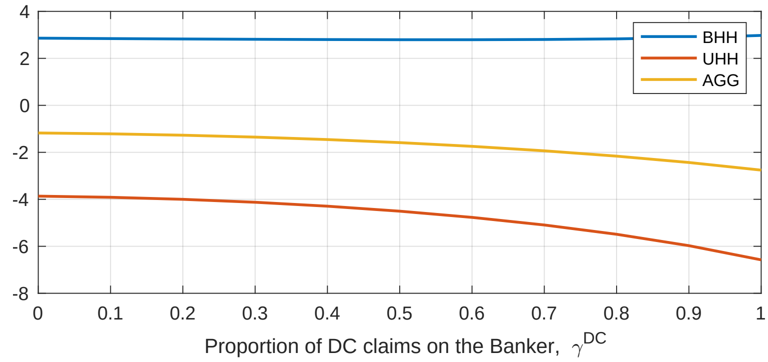
(a) Zero spread: $\delta^{DC} = 0$



(b) Positive spread: $\delta^{DC} = 0.05$



(c) Negative spread: $\delta^{DC} = -0.05$



Note: Plots summarises welfare for the BHH, UHH, and aggregate household as a function of the share of CBDC that are a source of funding on the bank balance sheet. Panels 11a, 11b, and 11c fix the spread between the policy rate and the CBDC rate at 0%, 5% and -5%, respectively. The welfare is calculated as a percent change from the regime with no digital currency for the retail direct (RD) and retail indirect (RI) schemes.

5 Conclusion

In this paper we focus on the financial inclusion effects of introducing a stable digital currency, namely a CBDC. We address a number of research questions as to the welfare implications of a retail CBDC design such as: differences between a direct claim held against a central bank verses an indirect scheme intermediated via private banks, the implications when interest rates on CBDCs are adjustable or fixed, and the strength of monetary policy transmission after CBDC adoption.

In the first part of this paper, we review the arguments for and against a retail CBDC using a simple endowment economy with two types of agents. Welfare for both sets of households improve with a retail CBDC, however we find the benefits are stronger for the unbanked. This supports the financial inclusion channel argument: unbanked households now have a savings device to smooth consumption and a buffer against macroeconomic fluctuations. We then extend the model to a New Keynesian set up with banks and financial frictions to examine the macroeconomic effects of issuing a digital currency. Similar to our endowment economy, we find the net benefit to a retail CBDC is stronger for unbanked households. This richer framework allows us to evaluate monetary policy rules and determine the magnitude of monetary policy transmission. Our results suggest that the introduction of retail CBDC amplifies monetary policy transmission to consumption. This is because the unbanked households now hold digital currency deposits and are sensitive to the central bank rate.

Next, we determine optimal monetary policy with two instruments: the policy rate on regular deposits and the rate on CBDC balances. This speaks to policy discussions on whether the CBDC rate should be adjustable or fixed, and whether monetary policy transmission requires CBDC deposits to be responsive to the policy rate. When CBDCs are a near-perfect substitute for bank deposits, we find optimal policy requires the CBDC rate to track the policy rate, yielding higher welfare than rules that require a constant rate of remuneration on the CBDC.

Our final contribution tests a number of elements of CBDC design: setting optimal taxes on the CBDC, the welfare effects of different spreads between the policy rate and the CBDC, and whether a CBDC should be intermediated through banks or through accounts held at the central bank. Based on the distributional effects of introducing a CBDC, we define a system of taxes and subsidies on the CBDC held by each household type. Specifically, taxing the banked households to subsidise unbanked households is strictly welfare improving in our setup. Considering a range of CBDC rates above and below the policy rate, we find that when the CBDC rate equals the policy rate aggregate welfare is maximised. Evaluating a CBDC intermediated through banks versus holding accounts directly with a central bank, we find the direct design is welfare improving when deposits are a more efficient base of funding for bank balance sheets than CBDC.

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A Appendix

A.1 Two-Agent Endowment Economy

A.1.1 Model without CBDC

The following equations outline a two-agent endowment economy model with no CBDCs. Defining the functional form for household utility as logarithmic in consumption,

$$u(C_t) = \ln C_t,$$

we define λ_t^i as the marginal utility of consumption of the type i household. Thus, our equilibrium conditions are:

Households.

$$\beta \mathbb{E}_t \frac{R_t}{\pi_{t+1}} \lambda_{t+1}^h = \lambda_t^h \tag{53}$$

$$\frac{1}{C_t^u} = \lambda_t^u + \mu_t^u \tag{54}$$

$$\beta \mathbb{E}_t \frac{\lambda_{t+1}^u + \mu_{t+1}^u}{\pi_{t+1}} = \lambda_t^u \left[1 + \phi_M (M_t - \bar{M})\right] \tag{55}$$

$$C_t^u + M_t = T_t^u + \frac{M_{t-1}}{\pi_t} \tag{56}$$

$$C_t^u = \frac{M_{t-1}}{\pi_t} \tag{57}$$

Market clearing.

$$0 = \Gamma_u (T_t^u - M_t) + \Gamma_h (T_t^h - C_t^h) \tag{58}$$

$$C_t = \Gamma_h C_t^h + \Gamma_u C_t^u \tag{59}$$

$$M_t = \frac{M_{t-1}}{\pi_t} \tag{60}$$

$$\omega_t = 1 - \frac{C_t^u}{C_t^h} \tag{61}$$

Exogenous processes.

$$\ln T_t^h = \rho_T \ln T_{t-1}^h + \varepsilon_t^T \tag{62}$$

$$\ln T_t^u = \rho_T \ln T_{t-1}^u + \varepsilon_t^T \tag{63}$$

A.1.2 Steady State for Endowment Economy without CBDC

Given that the steady state transfer amounts, $\bar{T}^i$, are exogenously set, the steady state is characterized by the following set of equations:

$$
\begin{aligned}
\bar{\pi} &= 1, \\
\bar{R} &= \frac{1}{\beta}, \\
\bar{M} &= \bar{T}^u_t, \\
\bar{\lambda}^u &= \beta, \\
\bar{\mu}^u &= 1-\beta, \\
\bar{C}^h &= \bar{T}^h, \\
\bar{C} &= \Gamma \bar{C}^h + (1-\Gamma)\bar{C}^u, \\
\bar{\omega} &= 1 - \frac{\bar{C}^u}{\bar{C}^h}.
\end{aligned}
$$

A.1.3 Model with CBDC

As above in Appendix A.1.1, the functional form for household utility is logarithmic in consumption. Thus, our equilibrium conditions are:

Households.

$$\frac{1}{C^h_t} = \lambda^h_t \tag{64}$$

$$\lambda^h_t = \beta \mathbb{E}_t \frac{R_t}{\pi_{t+1}} \lambda^h_{t+1} \tag{65}$$

$$\lambda^h_t \left(1 + \varkappa^{DC} DC^h_t\right) = \beta \frac{R^{DC}_t}{\pi_{t+1}} \lambda^h_{t+1} \tag{66}$$

$$\chi^{DC,h}_t = \frac{\varkappa^{DC}}{2} \left(DC^h_t\right)^2 \tag{67}$$

$$\frac{1}{C^u_t} = \lambda^u_t + \alpha_M \mu^u_t \tag{68}$$

$$\beta \mathbb{E}_t \frac{\lambda^u_{t+1} + \mu^u_{t+1}}{\pi_{t+1}} = \lambda^u_t \left[1 + \phi_M (M_t - \bar{M})\right] \tag{69}$$

$$\lambda^u_t (1 + \varkappa^{DC} DC^u_t) = \beta \mathbb{E}_t \frac{R^{DC}_t}{\pi_{t+1}} \lambda^u_{t+1} \tag{70}$$

$$\chi^{DC,u}_t = \frac{\varkappa^{DC}}{2} (DC^u_t)^2 \tag{71}$$

$$C^u_t + M_t + DC^u_t + \chi^{DC,u}_t = T^u_t + \frac{M_{t-1} + R^{DC}_{t-1} DC^u_{t-1}}{\pi_t} \tag{72}$$

$$\alpha_M C_t^u = \frac{M_{t-1}}{\pi_t} \tag{73}$$

Market clearing.

$$\Gamma_h T_t^h + \Gamma_u T_t^u = C_t + \Gamma_h \chi_t^{DC,h} + \Gamma_u \chi_t^{DC,u} \tag{74}$$

$$C_t = \Gamma_h C_t^h + \Gamma_u C_t^u \tag{75}$$

$$M_t = \frac{M_{t-1}}{\pi_t} \tag{76}$$

$$\omega_t = 1 - \frac{C_t^u}{C_t^h} \tag{77}$$

Exogenous processes.

$$\ln T_t^h = \rho_T \ln T_{t-1}^h + \varepsilon_t^T \tag{78}$$

$$\ln T_t^u = \rho_T \ln T_{t-1}^u + \varepsilon_t^T \tag{79}$$

A.1.4 Steady State for Endowment Economy with CBDC

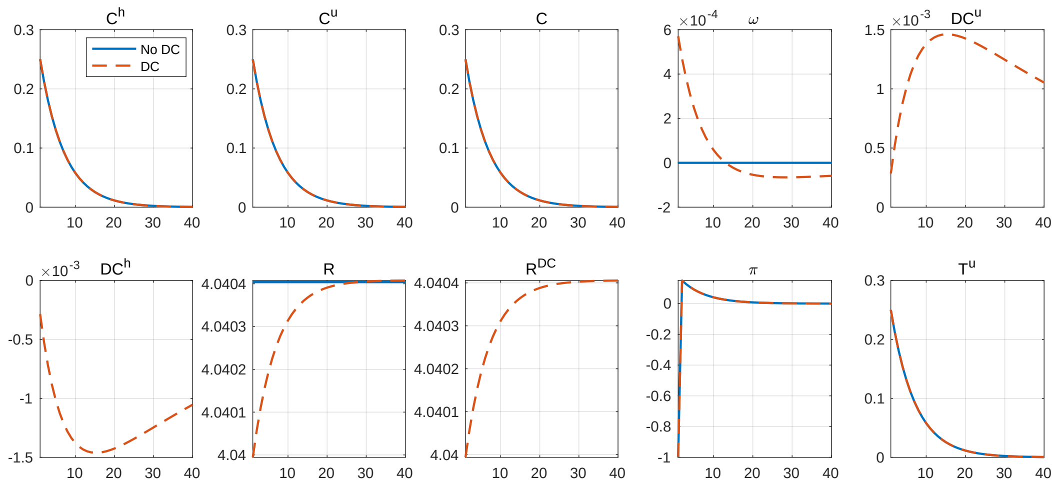
Given that the steady state transfer amounts, $\bar{T}^i$, are exogenously set, the steady state is characterized by the following set of equations:

$$\begin{aligned}
\bar{\pi} &= 1, \\
\bar{R} &= \frac{1}{\beta}, \\
\bar{DC}^h &= \bar{DC}^u = 0, \\
\bar{R}^{DC} &= \frac{1}{\beta} + \frac{1}{\beta}\varkappa^{DC}\bar{DC}^u, \\
\bar{C}^h &= \bar{T}^h + \bar{DC}^h\left(\bar{R}^{DC} - 1\right) - \bar{\chi}^{DC,h}, \\
\bar{C}^u &= \bar{T}^u + \bar{DC}^u\left(\bar{R}^{DC} - 1\right) - \bar{\chi}^{DC,u}, \\
\bar{M} &= \alpha_M \bar{C}^u, \\
\bar{\lambda}^u &= \frac{1}{\bar{C}^u\left(1 + \frac{\alpha_M}{\beta} - \alpha_M\right)}, \\
\bar{\mu}^u &= \bar{\lambda}^u\left(\frac{1}{\beta} - 1\right), \\
\bar{C} &= \Gamma_h \bar{C}^h + \Gamma_u \bar{C}^u, \\
\bar{\omega} &= 1 - \frac{\bar{C}^u}{\bar{C}^h}.
\end{aligned}$$

A.1.5 Endowment economy IRFs

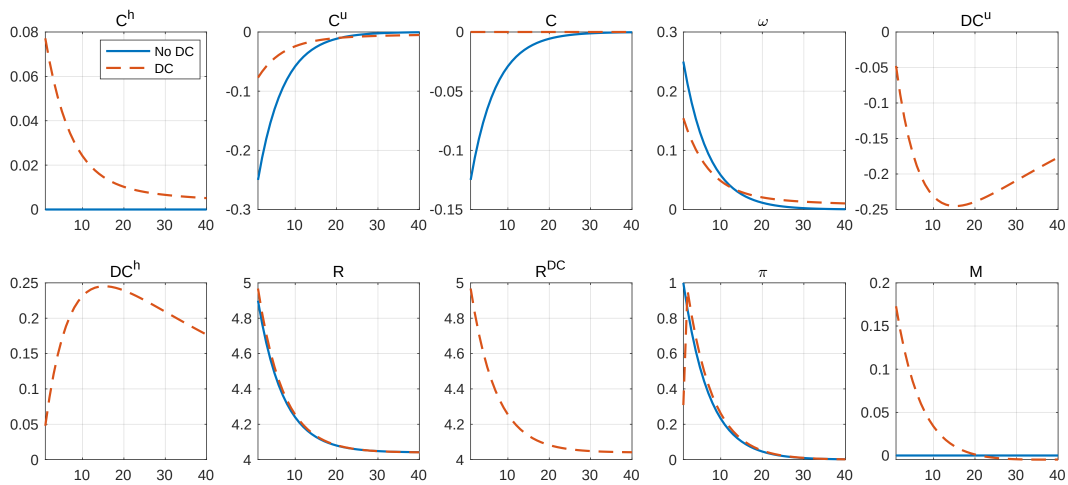
Below are the IRFs related to the endowment economy. We observe that presence of digital currency in the economy allows for better risk-sharing between the households. This is more vivid in the money supply shock case; as the money supply shock influences the UHH, they are eager to smooth consumption through borrowing in digital currency, which leads to perfect stabilization of aggregate consumption.

Figure 12: IRFs to 1% endowment shock



Note: Price variables are in levels, quantity variables are in percent deviations from steady state. The proportion of banked and unbanked households are set to a baseline value of $\Gamma_h = \Gamma_u = 0.5$.

Figure 13: IRFs to 1% money supply shock



Note: Price variables are in levels, quantity variables are in percent deviations from steady state. The proportion of banked and unbanked households are set to a baseline value of $\Gamma_h = \Gamma_u = 0.5$.

A.2 TANK model with Central Bank Digital Currency

A.2.1 The New Keynesian Phillips Curve

If we log linearize Equation (12) about the non-inflationary steady state, we yield the NKPC. First start by totally differentiating (12):

$$(2\bar{\pi} - 1)d\pi_t = \frac{\epsilon}{\kappa} dMC_t + \beta(2\bar{\pi} - 1)\mathbb{E}_t d\pi_{t+1},$$

where $\bar{\pi} = \bar{MC} = 1$ (recall that we have production subsidy τ to offset distortions arising from monopolistic competition). Substitute these values in and assume that $dMC_t = MC_t - \bar{MC}$ to get the log-linearized NKPC:

$$\hat{\pi}_t = \frac{\epsilon}{\kappa} \hat{MC}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1}, \tag{80}$$

where hatted variables denote log-deviations from steady state values (for any variable $x_t : \hat{x} = \ln \frac{x_t}{\bar{x}}$, and where we calibrate κ to a standard value as in, for example, Blanchard and Galí (2007):

$$\kappa = \frac{\epsilon\theta}{(1-\theta)(1-\beta\theta)}.$$

A.2.2 Household Optimisation Problem

The FOCs to the BHH problem are:

$$\lambda_t^h = \frac{1}{C_t^h + \zeta_0^h \frac{(L_t^h)^{1+\varsigma}}{1+\varsigma}}, \tag{81}$$

$$w_t = \zeta_0^h (L_t^h)^{\varsigma}, \tag{82}$$

$$1 = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{R_t}{\pi_{t+1}}, \tag{83}$$

$$1 = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{z_{t+1}^k + (1-\delta)Q_{t+1}}{Q_t + \varkappa^h \Gamma_h \left(\frac{K_t^h}{K_t}\right)}, \tag{84}$$

$$1 = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{R_t^{DC}}{\pi_{t+1}(1 + \varkappa^{DC} DC_t^h)}. \tag{85}$$

The FOCs to the UHH problem are:

$$\lambda_t^u + \alpha_M \mu_t^u = \frac{1}{C_t^u + \zeta_0^u \frac{(L_t^u)^{1+\varsigma}}{1+\varsigma}}, \tag{86}$$

$$\lambda_t^u w_t = \frac{\zeta_0^u}{C_t^u - \zeta_0^u \frac{(L_t^u)^{1+\varsigma}}{1+\varsigma}} (L_t^u)^{\varsigma}, \tag{87}$$

$$\lambda_t^u \left[1 + \phi_M(M_t - \bar{M})\right] = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}^u + \mu_{t+1}^u}{\pi_{t+1}}\right], \tag{88}$$

$$1 = \beta \mathbb{E}_t \frac{\lambda_{t+1}^u}{\lambda_t^u} \frac{R_t^{DC}}{\pi_{t+1}(1 + \varkappa^{DC} DC_t^u)}. \tag{89}$$

A.2.3 Rewriting the Banker's Problem

To setup the problem of the banker as in Section 3.3.2, first iterate the banker's flow of funds constraint (25) forward by one period, and then divide through by n_t to yield:

$$\frac{n_{t+1}}{n_t} = \frac{\left(z_{t+1}^k + (1-\delta)Q_{t+1}\right)}{Q_t} \frac{Q_t k_t^b}{n_t} - \frac{R_t}{\pi_{t+1}} \frac{d_t}{n_t} - \frac{R_t^{DC}}{\pi_{t+1}} \frac{dc_t}{n_t}.$$

Rearrange the balance sheet constraint (24) and use the fact that $dc_t/n_t = x_t\phi_t$, to yield the following:

$$\frac{d_t}{n_t} = \frac{\varkappa^b}{2} x_t^2 \phi_t + \phi_t - x_t\phi_t - 1.$$

Substitute this value for d_t/n_t into the expression for n_{t+1}/n_t, and we get:

$$\frac{n_{t+1}}{n_t} = \left(\frac{z_{t+1}^k + (1-\delta)Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}}\right)\phi_t + \left(\frac{R_t}{\pi_{t+1}} - \frac{R_t^{DC}}{\pi_{t+1}}\right) x_t\phi_t + \left(1 - \frac{\varkappa^b}{2} x_t^2 \phi_t\right) \frac{R_t}{\pi_{t+1}}.$$

Substituting this expression into (26), yields the following:

$$\begin{aligned}
\psi_t &= \mathbb{E}_t \left[\Lambda_{t,t+1}^h (1 - \sigma_b + \sigma_b \psi_{t+1}) \left\{ \begin{array}{c} \left(\frac{z_{t+1}^k + (1-\delta)Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}}\right)\phi_t \\ + \left(\frac{R_t}{\pi_{t+1}} - \frac{R_t^{DC}}{\pi_{t+1}}\right) x_t\phi_t \\ + \left(1 - \frac{\varkappa^b}{2} x_t^2\phi_t\right) \frac{R_t}{\pi_{t+1}} \end{array} \right\} \right] \\
&= \mu_t\phi_t + \mu_t^{DC} x_t\phi_t + \left(1 - \frac{\varkappa^b}{2} x_t^2 \phi_t\right) \upsilon_t,
\end{aligned}$$

which is (3.3.2) in the text.

A.2.4 Solving the banker's problem

With $\{\mu_t, \mu_t^{DC}\} > 0$, the banker's incentive compatibility constraint binds with equality, and so we can write the Lagrangian as:

$$\mathcal{L} = \psi_t + \lambda_t(\psi_t - \theta^b \phi_t),$$

where λ_t is the Lagrangian multiplier. The FOCs are:

$$(1 + \lambda_t)\left[\mu_t + \mu_t^{DC} x_t - \frac{\varkappa^b}{2} x_t^2 \upsilon_t\right] = \lambda_t \theta^b, \tag{90}$$

$$\psi_t = \theta^b \phi_t. \tag{91}$$

Substitute (91) into the banker's objective function to yield:

$$\phi_t = \frac{\upsilon_t}{\theta^b - \mu_t - \mu_t^{DC} x_t + \frac{\varkappa^b}{2} x_t^2 \upsilon_t}, \tag{92}$$

which is (29) in the text.

A.2.5 Full Set of Equilibrium Conditions

Households.

$$w_t = \zeta_0^h L_t^h \tag{93}$$

$$1 = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{R_t}{\pi_{t+1}} \tag{94}$$

$$1 = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{z_{t+1}^k + (1-\delta) Q_{t+1}}{Q_t + \varkappa^h \Gamma_h \left(\frac{K_t^h}{K_t}\right)} \tag{95}$$

$$1 = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{R_t^{DC}}{\pi_{t+1}(1 + \varkappa^{DC} DC_t^h)} \tag{96}$$

$$C_t^u + M_t + \chi_t^M + DC_t^u + \chi_t^{DC,u} + T_t^u = w_t L_t^u + \frac{M_{t-1}}{\pi_t} + \frac{R_{t-1}^{DC}}{\pi_t} DC_{t-1}^u \tag{97}$$

$$\frac{\lambda_t^u}{\lambda_t^u + \alpha_M \mu_t^u} w_t = \zeta_0^u (L_t^u)^\zeta \tag{98}$$

$$\lambda_t^u + \alpha_M \mu_t^u = \frac{1}{C_t^u + \zeta_0^u \frac{(L_t^u)^{1+\zeta}}{1+\zeta}} \tag{99}$$

$$\beta \mathbb{E}_t \frac{\lambda_{t+1}^u + \mu_{t+1}^u}{\pi_{t+1}} = \lambda_t^u \left[1 + \phi_M (M_t - \bar{M})\right] \tag{100}$$

$$\lambda_t^u (1 + \varkappa^{DC} DC_t^u) = \beta \mathbb{E}_t \lambda_{t+1}^u \frac{R_t^{DC}}{\pi_{t+1}} \tag{101}$$

$$\alpha_M C_t^u = \frac{M_{t-1}}{\pi_t} \tag{102}$$

Production.

$$Q_t = 1 + \frac{\kappa_I}{2} \left(\frac{I_t}{\bar{I}}\right)^2 - \frac{\kappa_I}{\bar{I}} \left(\frac{I_t}{\bar{I}} - 1\right) \tag{103}$$

$$K_t = (1-\delta) K_{t-1} + I_t \tag{104}$$

$$Y_t = A_t K_{t-1}^\alpha L_t^{1-\alpha} \tag{105}$$

$$w_t = (1-\alpha) A_t \left(\frac{K_{t-1}}{L_t}\right)^\alpha \tag{106}$$

$$MC_t = \frac{1}{A_t}\left(\frac{z_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{107}$$

$$\begin{aligned}\pi_t(\pi_t - 1) = {} & \frac{1}{\kappa}\left[\epsilon MC_t + 1 - \epsilon + \tau\epsilon - \tau\right] \\ & + \mathbb{E}_t\left[\Lambda^h_{t,t+1}(\pi_{t+1}-1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] + \xi_t^{\pi}\end{aligned} \tag{108}$$

Banks.

$$\psi_t = \theta^b \phi_t \tag{109}$$

$$\phi_t = \frac{\upsilon_t}{\theta^b - \mu_t - \mu_t^{DC} x_t + \frac{\varkappa^b}{2} x_t^2 \upsilon_t} \tag{110}$$

$$\mu_t = \mathbb{E}_t\left[\Omega_{t,t+1}\left\{\frac{z^k_{t+1} + (1-\delta)Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}}\right\}\right] \tag{111}$$

$$\mu_t^{DC} = \mathbb{E}_t\left[\Omega_{t,t+1}\left\{\frac{R_t}{\pi_{t+1}} - \frac{R_t^{DC}}{\pi_{t+1}}\right\}\right] \tag{112}$$

$$\upsilon_t = \mathbb{E}_t\left[\Omega_{t,t+1}\frac{R_t}{\pi_{t+1}}\right] \tag{113}$$

$$\Omega_{t,t+1} = \Lambda^h_{t,t+1}(1 - \sigma_b + \sigma_b \psi_{t+1}) \tag{114}$$

Monetary and fiscal policy.

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_R}\left(\pi_t^{\phi_\pi} X_t^{\phi_Y}\right)^{1-\rho_R}\exp(\varepsilon_t^R) \tag{115}$$

$$-\tau Y_t = \Gamma_h T_t^h + \Gamma_u T_t^u \tag{116}$$

$$R_t^{DC} = R_t \tag{117}$$

Market clearing.

$$C_t = \Gamma_h C_t^h + \Gamma_u C_t^u \tag{118}$$

$$L_t = \Gamma_h L_t^h + \Gamma_u L_t^u \tag{119}$$

$$DC_t = \Gamma_h DC_t^h + \Gamma_u DC_t^u \tag{120}$$

$$\begin{aligned}Y_t = {} & C_t + \left[1 + \Phi\left(\frac{I_t}{\bar{I}}\right)\right] I_t + \frac{\kappa}{2}(\pi_t - 1)^2 Y_t \\ & + \Gamma_h(\chi_t^h + \chi_t^b + \chi_t^{DC,h}) + \Gamma_u(\chi_t^M + \chi_t^{DC,u})\end{aligned} \tag{121}$$

$$\begin{aligned}Y_t^f = {} & C_t + \left[1 + \Phi\left(\frac{I_t}{\bar{I}}\right)\right] I_t \\ & + \Gamma_h(\chi_t^h + \chi_t^b + \chi_t^{DC,h}) + \Gamma_u(\chi_t^M + \chi_t^{DC,u})\end{aligned} \tag{122}$$

$$X_t = \frac{Y_t}{Y_t^f} \tag{123}$$

$$K_t = \Gamma_h(K_t^h + K_t^b) \tag{124}$$

$$\begin{aligned} N_t = \sigma_b \left[(z_t^k + (1-\delta)Q_t)K_{t-1}^b - \frac{R_{t-1}}{\pi_t} D_{t-1} - \frac{R_{t-1}^{DC}}{\pi_t} \frac{DC_{t-1}}{\Gamma_h} \right] \\ + \gamma_b(z_t^k + (1-\delta)Q_t)\frac{K_{t-1}}{\Gamma_h} \end{aligned} \tag{125}$$

$$Q_t K_t^b = \phi_t N_t \tag{126}$$

$$\left(1 + \frac{\varkappa^b}{2} x_t^2\right) Q_t K_t^b = D_t + \frac{DC_t}{\Gamma_h} + N_t \tag{127}$$

$$x_t = \frac{DC_t}{Q_t \Gamma_h K_t^b} \tag{128}$$

Exogenous processes.

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A \tag{129}$$

$$\xi_t^\pi = \rho_\pi \xi_{t-1}^\pi + \varepsilon_t^\pi \tag{130}$$

A.2.6 Model Steady State

In the non-stochastic steady state, we have the following:

$$\begin{aligned} \bar{Q} &= 1, \\ \bar{\pi} &= 1, \\ \bar{R} &= \frac{1}{\beta}, \\ \bar{R}^{DC} &= \bar{R}. \end{aligned}$$

We define the discounted spreads on equity and DC as:

$$s = \beta[\bar{z}^k + (1-\delta)] - 1, \tag{131}$$

$$s^{DC} = 1 - \beta \bar{R}^{DC} = 0, \tag{132}$$

which we consider to be endogenous and exogenous, respectively.[15]

15. Note that we could have a non-zero discounted spread between the return on digital currency and the deposit.

From the BHH's FOC with respect to equity, (84), we have:

$$
\begin{aligned}
1 &= \beta\left[\frac{\bar{z}^k + (1-\delta)}{1 + \varkappa^h \Gamma_h \frac{\bar{K}^h}{\bar{K}}}\right] \\
1 + \varkappa^h \Gamma_h \frac{\bar{K}^h}{\bar{K}} &= \beta\left[\bar{z} + (1-\delta)\right] \\
\Gamma_h \frac{\bar{K}^h}{\bar{K}} &= \frac{s}{\varkappa^h}.
\end{aligned}
\tag{133}
$$

Additionally, in steady state we have:

$$
\begin{aligned}
\bar{\Omega} &= \beta(1 - \sigma_b + \sigma_b \bar{\psi}), \\
\bar{\upsilon} &= \frac{\bar{\Omega}}{\beta}, \\
\bar{\mu} &= \bar{\Omega}\left[\bar{z}^k + (1-\delta) - \frac{1}{\beta}\right], \\
\bar{\mu}^{DC} &= \bar{\Omega}\left[\frac{1}{\beta} - \bar{R}^{DC}\right],
\end{aligned}
$$

and so, using (131) and (132), we can write:

$$
\begin{aligned}
\frac{\bar{\mu}}{\bar{\upsilon}} &= s, \\
\frac{\bar{\mu}^{DC}}{\bar{\upsilon}} &= s^{DC} \implies \bar{\mu}^{DC} = 0.
\end{aligned}
$$

Next, define J as:

$$
J = \frac{n_{t+1}}{n_t} = \left[\bar{z}^k + (1-\delta)\right]\frac{\bar{K}^b}{\bar{N}} - \bar{R}\frac{\bar{D}}{\bar{N}} - \bar{R}^{DC}\frac{\bar{DC}}{\Gamma_h \bar{N}},
$$

and use the following:

$$
\begin{aligned}
\frac{\bar{D}}{\bar{N}} &= \frac{\varkappa^b}{2}\bar{\phi}\bar{x}^2 + \bar{\phi} - \bar{x}\bar{\phi} - 1, \\
\bar{\phi} &= \frac{\bar{K}^b}{\bar{N}}, \\
\frac{\bar{DC}}{\Gamma_h \bar{N}} &= \bar{\phi}\bar{x},
\end{aligned}
$$

to write J as:

$$\begin{aligned}J &= (\bar{z}^k + (1-\delta) - \bar{R})\bar{\phi} + \left(1 - \frac{\varkappa^b}{2}\bar{x}^2\bar{\phi}\right)\bar{R} + (\bar{R} - \bar{R}^{DC})\bar{x}\bar{\phi} \\ &= \frac{1}{\beta}\left[p(s, s^{DC})\bar{\phi} + 1\right],\end{aligned}$$

where

$$p(s, s^{DC}) \equiv s + s^{DC}\bar{x} - \frac{\varkappa^b}{2}\bar{x}^2$$

is defined as the return premium.

Then, from (42) we have:

$$\begin{aligned}\bar{N} &= \sigma_b\left\{\left[\bar{z}^k + (1-\delta)\right]\bar{K}^b - \bar{R}\bar{D} - \bar{R}^{DC}\frac{\bar{DC}}{\Gamma}\right\} + \gamma_b\left[\bar{z}^k + (1-\delta)\right]\frac{\bar{K}}{\Gamma} \\ \frac{\bar{N}}{\bar{N}} &= \sigma_b\left\{\left[\bar{z}^k + (1-\delta)\right]\frac{\bar{K}^b}{\bar{N}} - \bar{R}\frac{\bar{D}}{\bar{N}} - \bar{R}^{DC}\frac{\bar{DC}}{\Gamma\bar{N}}\right\} + \frac{\gamma_b}{\bar{N}}\left[\bar{z}^k + (1-\delta)\right]\frac{\bar{K}}{\Gamma} \\ \beta &= \sigma_b\beta J + \frac{\gamma_b}{\bar{N}}\beta\left[\bar{z}^k + (1-\delta)\right]\frac{\bar{K}}{\Gamma} \\ &= \sigma_b\beta J + \frac{\gamma_b\bar{K}^b}{\bar{N}}\left(1 + \varkappa^h\Gamma\frac{\bar{K}^h}{\bar{K}}\right)\frac{\bar{K}}{\Gamma\bar{K}^b} \\ &= \sigma_b\beta J + \gamma_b(1+s)\bar{\phi}\frac{1}{\frac{\Gamma\bar{K}^b}{\bar{K}}} \\ &= \sigma_b\beta J + \gamma_b(1+s)\bar{\phi}\frac{1}{\frac{\bar{K} - \Gamma\bar{K}^h}{\bar{K}}} \\ &= \sigma_b\left[p(s, s^{DC})\bar{\phi} + 1\right] + \gamma_b(1+s)\bar{\phi}\frac{1}{1 - \frac{s}{\varkappa^h}} \\ \beta &= \sigma_b + \left[\sigma_b p(s, s^{DC}) + \gamma_b\frac{1+s}{1 - \frac{s}{\varkappa^h}}\right]\bar{\phi},\end{aligned}$$

or

$$\bar{\phi} = \frac{\beta - \sigma_b}{\sigma_b p(s, s^{DC}) + \gamma_b\frac{1+s}{1 - \frac{s}{\varkappa^h}}}$$

Equation (26) in steady state gives us:

$$
\begin{aligned}
\bar{\psi} &= \beta(1-\sigma_b+\sigma_b\bar{\psi})J \\
&= \beta J - \beta\sigma_b J + \beta\sigma_b\bar{\psi}J \\
&= \beta(1-\sigma_b)J + \beta\sigma_b\bar{\psi}J \\
&= \frac{\beta(1-\sigma_b)J}{1-\beta\sigma_b J} \\
&= \frac{(1-\sigma_b)\left[p(s,s^{DC})\bar{\phi}+1\right]}{1-\sigma_b\left[p(s,s^{DC})\bar{\phi}+1\right]} \\
&= \frac{(1-\sigma_b)\left[p(s,s^{DC})\bar{\phi}+1\right]}{1-\sigma_b-\sigma_b p(s,s^{DC})\bar{\phi}},
\end{aligned}
$$

and from (91) we have

$$\bar{\psi} = \theta^b\bar{\phi}.$$

Combine the expressions for $\bar{\phi}$ and $\bar{\psi}$ to get:

$$
\frac{\theta^b(\beta-\sigma_b)}{\sigma_b p(s,s^{DC})+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}} = \frac{(1-\sigma_b)\left[\frac{p(s,s^{DC})(\beta-\sigma_b)}{\sigma_b p(s,s^{DC})+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}}+1\right]}{1-\sigma_b-\sigma_b\left[\frac{p(s,s^{DC})(\beta-\sigma_b)}{\sigma_b p(s,s^{DC})+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}}\right]},
$$

then rearrange:

$$
\begin{aligned}
0 &= H(s,s^{DC}) \\
&= (1-\sigma_b)\left[\beta p(s,s^{DC})+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}\right]\left[\sigma_b p(s,s^{DC})+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}\right] \\
&\quad -\theta^b(\beta-\sigma_b)\left[\sigma_b(1-\beta)p(s,s^{DC})+(1-\sigma_b)\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}\right].
\end{aligned}
$$

In the absence of any taxes or subsidies on digital currency deposits, and in the case where $s^{DC}=\bar{\mu}^{DC}=0$, the fixed management cost of digital currency $\varkappa^{DC}>0$ imply that $\bar{DC}=\bar{x}=0$. Thus, we can write the risk premium as:

$$p(s,0)\to s,$$

and

$$
\begin{aligned}
H(s,0) = (1-\sigma_b)\left[s\beta + \gamma_b \frac{1+s}{1-\frac{s}{\varkappa^h}}\right]\left[s\sigma_b + \gamma_b \frac{1+s}{1-\frac{s}{\varkappa^h}}\right] \\
- \theta^b(\beta - \sigma_b)\left[\sigma_b(1-\beta)s + (1-\sigma_b)\gamma_b \frac{1+s}{1-\frac{s}{\varkappa^h}}\right].
\end{aligned}
$$

We can observe that as $\gamma_b \to 0$,

$$
\begin{aligned}
H(s,0) &= (1-\sigma_b)s^2\beta\sigma_b - \theta^b(\beta-\sigma_b)\left[\sigma_b(1-\beta)s\right] \\
\implies s &\to \theta^b \frac{(\beta-\sigma_b)(1-\beta)}{(1-\sigma_b)\beta}.
\end{aligned}
$$

Thus, there exists a unique steady state equilibrium with positive spread $s > 0$ for a small enough γ_b.

Given s, we then yield:

$$
\bar{z}^k = \frac{1}{\beta}(1+s) - (1-\delta),
$$

and since in the steady state $\bar{MC} = 1$, we also have:

$$
\begin{aligned}
\bar{MC} &= \left(\frac{\bar{z}^k}{\alpha}\right)^{\alpha}\left(\frac{\bar{w}}{1-\alpha}\right)^{1-\alpha} = 1 \\
&= \left(\frac{\bar{Y}}{\bar{K}}\right)^{\alpha}\left(\frac{\bar{Y}}{\bar{L}}\right)^{1-\alpha} \\
&= \left(\frac{\bar{Y}}{\bar{K}}\right)^{\alpha}\left(\frac{\bar{z}^k}{\alpha}\right)^{\frac{\alpha(1-\alpha)}{\alpha-1}},
\end{aligned}
$$

with

$$
\begin{aligned}
\bar{w} &= (1-\alpha)\frac{\bar{Y}}{\bar{L}}, \\
\bar{z}^k &= \alpha\frac{\bar{Y}}{\bar{K}}, \\
\implies \bar{w} &= (1-\alpha)\left(\frac{\bar{z}^k}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}.
\end{aligned}
$$

Put these together to get:

$$
\begin{aligned}
\frac{\bar{K}}{\bar{Y}} &= \left(\frac{\bar{z}^k}{\alpha}\right)^{\frac{1-\alpha}{\alpha-1}} \\
&= \frac{\alpha}{\bar{z}^k}.
\end{aligned}
$$

From the FOCs of the BHH and UHH problem, we have:

$$\bar{w} = \zeta_0^h (\bar{L}^h)^{\zeta},$$
$$\bar{w} = \frac{\zeta_0^u (\bar{L}^u)^{\zeta} (1 + \frac{\alpha_M}{\beta} - \alpha_M)}{\left[C^u - \zeta_0^u \frac{(L^u)^{1+\zeta}}{1+\zeta}\right]}.$$

But since we have that $\zeta_0^u = \frac{\zeta_0^h}{(1+\frac{\alpha_M}{\beta}-\alpha_M)}$, we can write:

$$\bar{w} = \zeta_0^h \bar{L}^{\zeta}.$$

We can then use our previous expression for $\bar{w}$ to express $\bar{L}$ as a function of $\bar{z}^k$:

$$\bar{L} = \left[\frac{1-\alpha}{\zeta_0^h} \left(\frac{\bar{z}^k}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{1}{\zeta}}.$$

Since we know that

$$\bar{w} = (1-\alpha)\frac{\bar{Y}}{\bar{L}},$$

we yield:

$$\bar{Y} = \frac{\zeta_0^h}{\alpha} \left[\frac{1-\alpha}{\zeta_0^h} \left(\frac{\bar{z}^k}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{1+\zeta}{\zeta}}.$$

Additionally, we have:

$$\frac{\bar{I}}{\bar{K}} = \delta,$$

and

$$\frac{1}{\beta} = \frac{\alpha\frac{\bar{Y}}{\bar{K}} + 1 - \delta}{1 + \varkappa^h \Gamma_h \frac{\bar{K}^h}{\bar{K}}}$$
$$\Leftrightarrow \frac{\bar{Y}}{\bar{K}} = \frac{\beta^{-1}(1+s) + \delta - 1}{\alpha},$$

from (133), and:

$$\frac{\bar{I}}{\bar{Y}} = \frac{\bar{I}/\bar{K}}{\bar{Y}/\bar{K}} = \frac{\alpha\delta}{\beta^{-1}(1+s) + \delta - 1}.$$

These of course imply:

$$\bar{K} = \left[\frac{1-\alpha}{\zeta_0^h} \left(\frac{\bar{z}^k}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{1+\zeta}{\zeta}} \frac{\zeta_0^h}{\beta^{-1}(1+s) + \delta - 1}$$

With $\bar{K}$ and s in hand, we can then turn back to the BHH's FOC wrt to equity, (84), to find $\bar{K}^h$:

$$\bar{K}^h = \frac{s}{\varkappa^h}\frac{\bar{K}}{\Gamma_h},$$

and also get $\bar{K}^b$:

$$\bar{K}^b = \frac{\bar{K}}{\Gamma_h} - \bar{K}^h.$$

This then gives us $\bar{N}$ as we already solved $\bar{\phi}$:

$$\bar{N} = \frac{\bar{K}^b}{\bar{\phi}}.$$

Then $\bar{D}$ is also solved as a residual from (24):

$$\bar{D} = \bar{K}^b - \bar{N}.$$

Given $\bar{Y}$, $\bar{I}$, and $\bar{K}$, we can get $\bar{C}$:

$$\frac{\bar{C}}{\bar{Y}} = 1 - \frac{\bar{I}}{\bar{Y}} - \frac{\varkappa^h}{2}(\Gamma_h \bar{K}^h)^2 \left(\frac{\bar{K}}{\bar{Y}}\right)^{-1}.$$

From the UHH's FOC with respect to M, we have:

$$\bar{\mu}^u = \bar{\lambda}^h \left(\frac{1}{\beta} - 1\right),$$

and the FOC with respect to consumption gives us an expression for the marginal utility from consumption:

$$\bar{\lambda}^h \left(1 + \frac{\alpha_M}{\beta} - \alpha_M\right).$$

Thus, we can express $\bar{\lambda}^u$ as a function of the marginal utility from consumption:

$$\frac{1}{\bar{\lambda}^u} = \left(1 + \frac{\alpha_M}{\beta} - \alpha_M\right)\left(\bar{C}^u - \zeta_0^u \frac{(\bar{L}^u)^{1+\zeta}}{1+\zeta}\right),$$

noting that because of the values of ζ_0^h and ζ_0^u, we have:

$$\bar{L}^u = \left(\frac{\bar{w}}{\zeta_0^h}\right)^{\frac{1}{\zeta}}.$$

Finally, much like aggregate digital currency holdings, the BHH will not hold any digital currency holdings in steady state due to the presence of management costs. This means that in steady state:

$$\bar{DC}^h = 0,$$

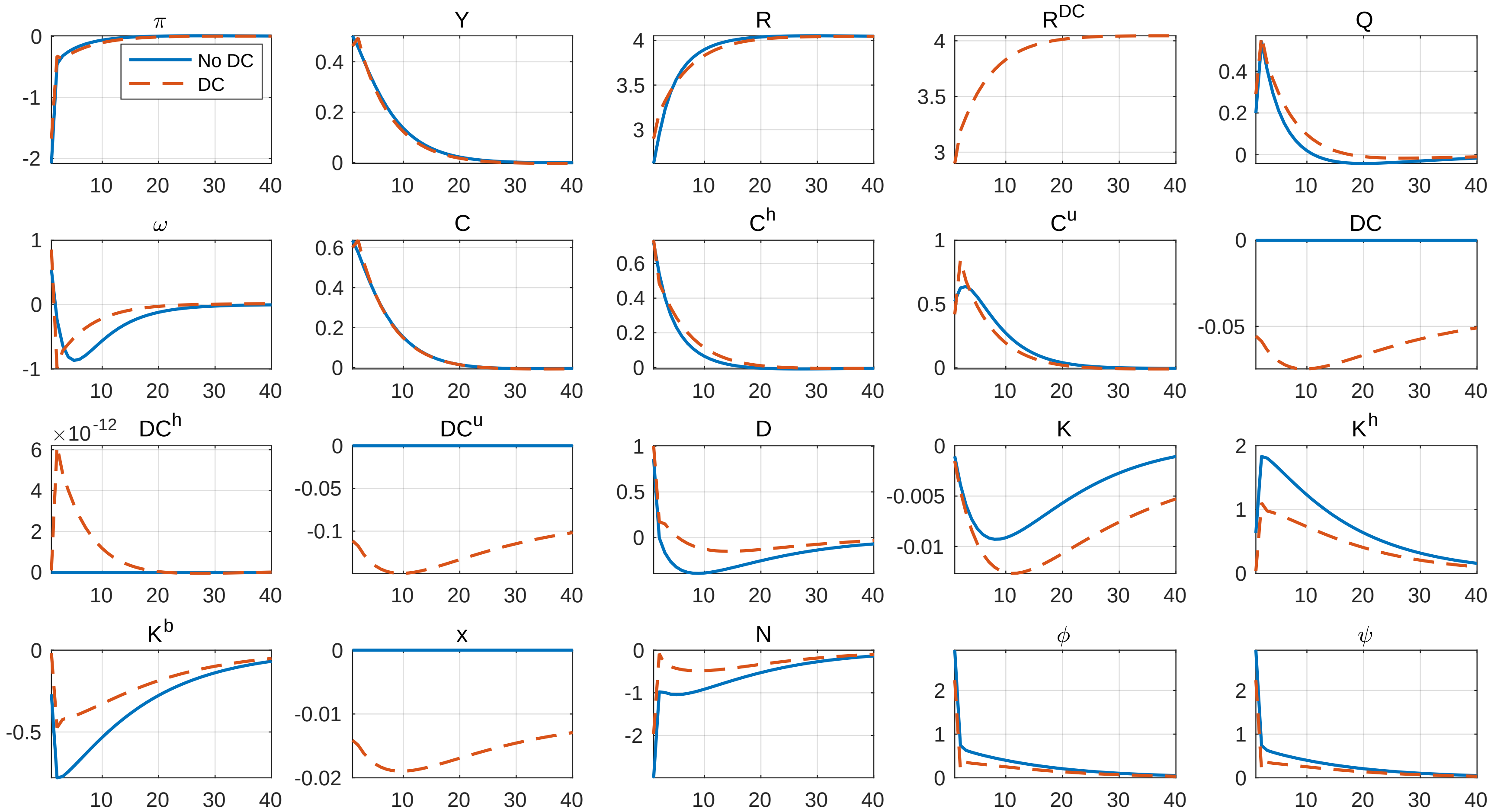
which, of course, implies:

$$\bar{DC}^u = 0.$$

A.2.7 Fundamental-Based Shocks

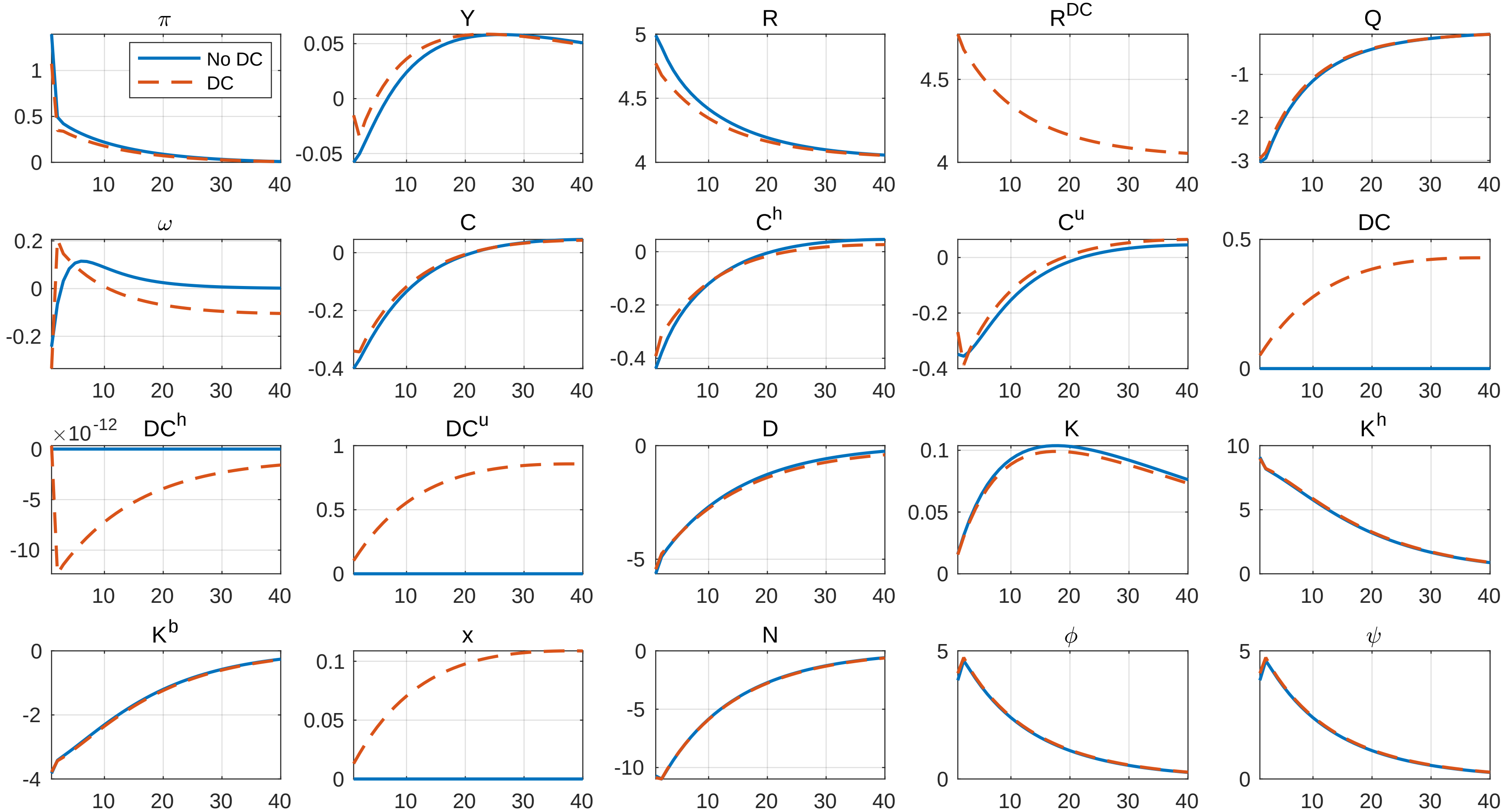
Figures 14 and 15 present results in response to a 1 basis point TFP shock. The addition of a CBDC increases the sensitivity of consumption of UHH to fundamental-based shocks. The intuition is as follows: the addition of a savings vehicle allows the UHH to change consumption more aggressively in response to positive shocks to TFP.

Figure 14: IRFs to a 1% ann. TFP shock



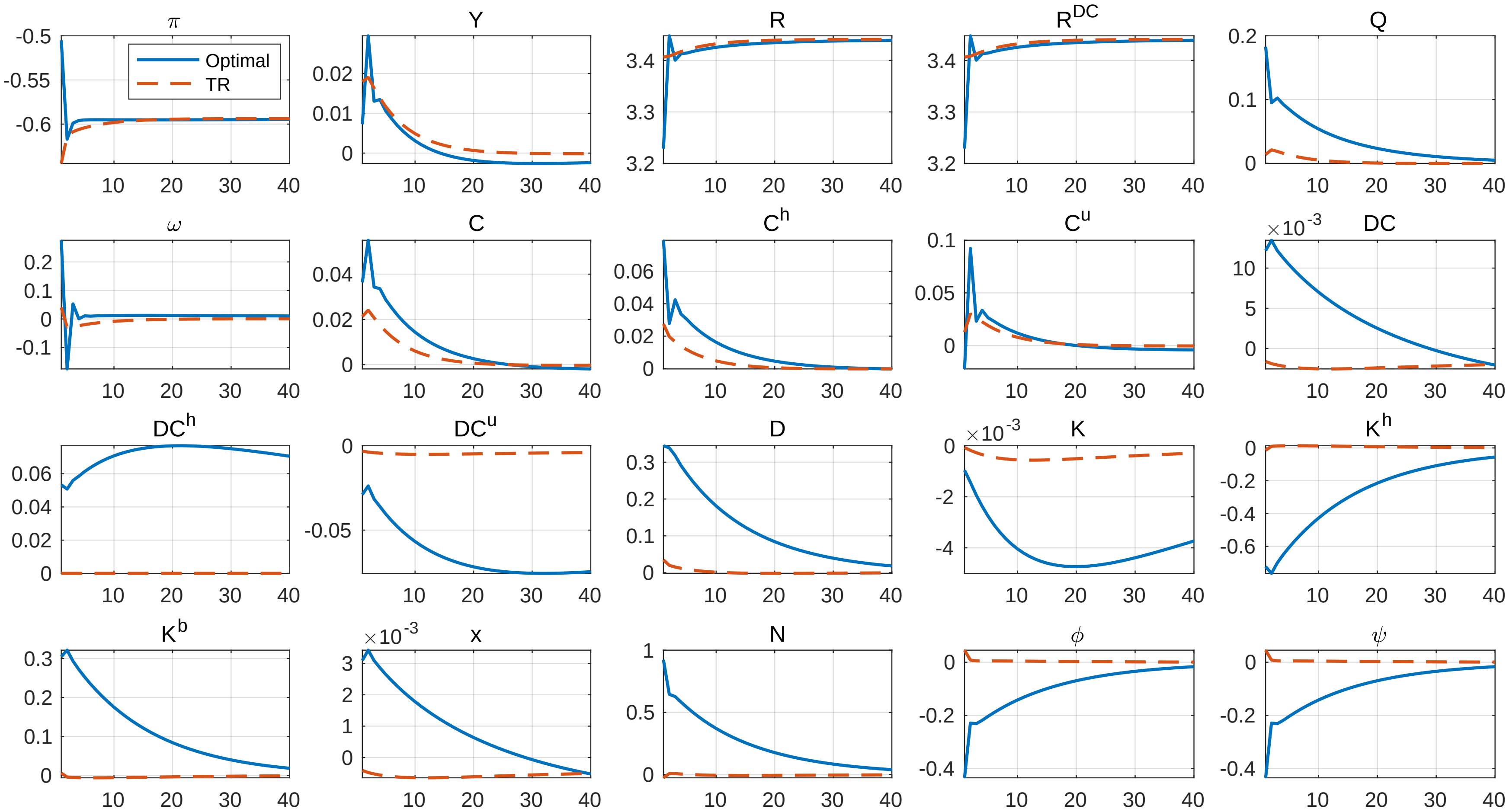
Note: Figure plots impulse responses of model variables with respect to a 1 % annualized innovation to TFP. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R) and Digital Currency Returns (R^{DC}) which are expressed as annualised net rates.

Figure 15: IRFs to a 1 % ann. cost-push shock



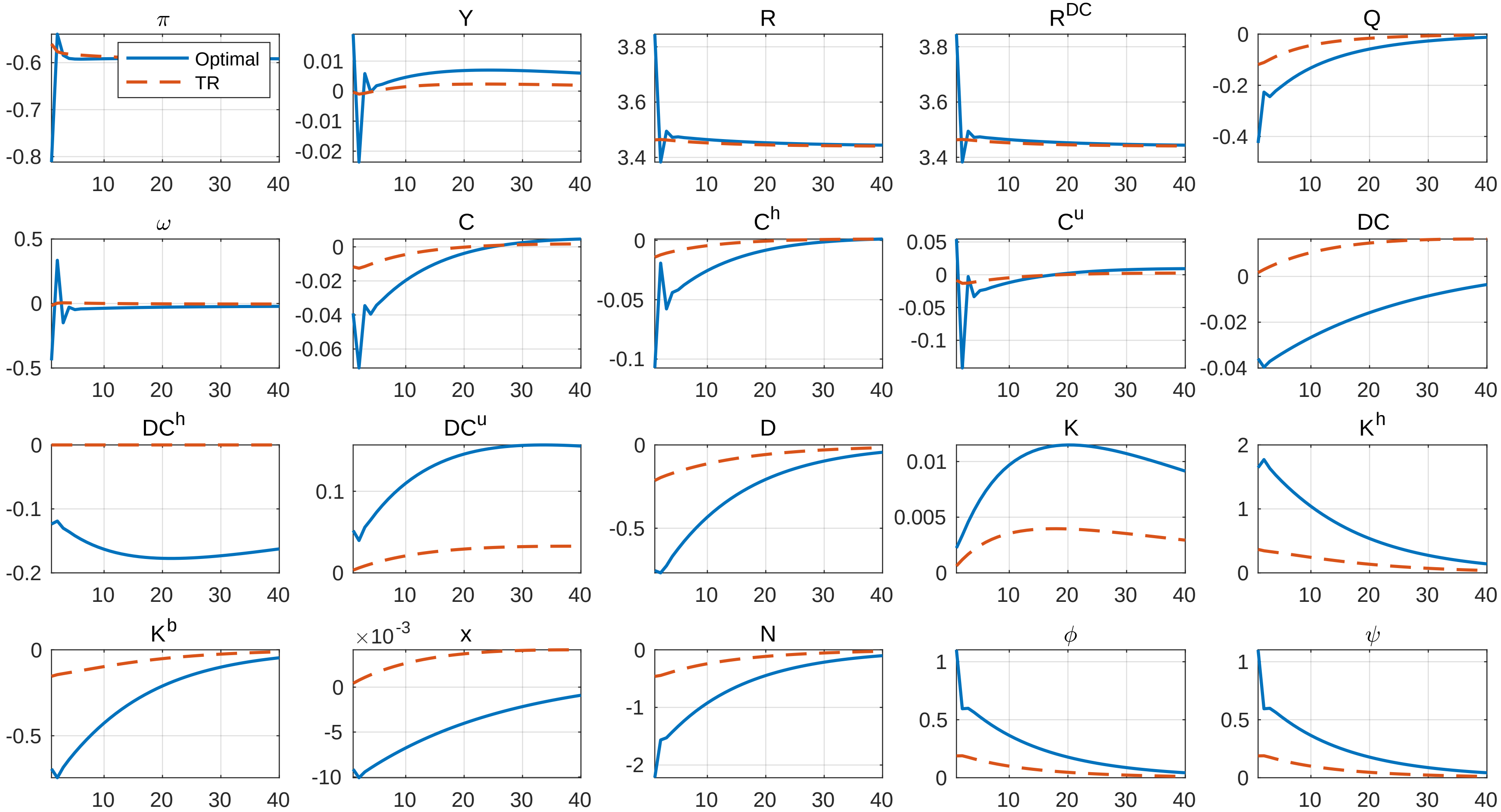
Note: Figure plots impulse responses of model variables with respect to a 1 % annualized cost-push shock. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R) and Digital Currency Returns (R^{DC}) which are expressed as annualised net rates.

Figure 16: Optimal policy IRFs to a 1 basis point TFP shock



Note: Figure plots impulse responses of model variables with respect to a 1 basis point innovation to TFP. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R) and Digital Currency Returns (R^{DC}) which are expressed as annualised net rates.

Figure 17: Optimal policy IRFs to a 1 basis point cost-push shock



Note: Figure plots impulse responses of model variables with respect to a 1 basis point cost-push shock. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R) and Digital Currency Returns (R^{DC}) which are expressed as annualised net rates.

A.2.8 Impact of Digital Currency Adjustment Costs

As noted in Section 3.7, as the cost parameter of digital currency holdings, $\varkappa^{DC} \to 0$ – which can be thought of as the degree of imperfection of digital currency relative to deposits – deposits and digital currency become perfect substitutes and, thus, the importance of R^{DC} as a distinct policy instrument is attenuated. We perform sensitivity analysis to show how imperfections in design of digital currency influence optimal policy.

In Figures 18 and 19 we illustrate the dynamics of the model with respect to a TFP shock and cost-push shock, respectively, for different levels of $\varkappa^{DC}$. We observe that a higher degree of imperfection of digital currency design leads to two distinct consequences: (i) potency of monetary policy to reduce variance of inflation and output is attenuated, (ii) optimal spread between R and R^{DC} is higher. A higher degree of imperfection of digital currency has direct effect on consumer welfare as the households face higher digital currency adjustment costs.

Figure 18: IRFs to a 1bp TFP shock (low and high $\varkappa^{DC}$)

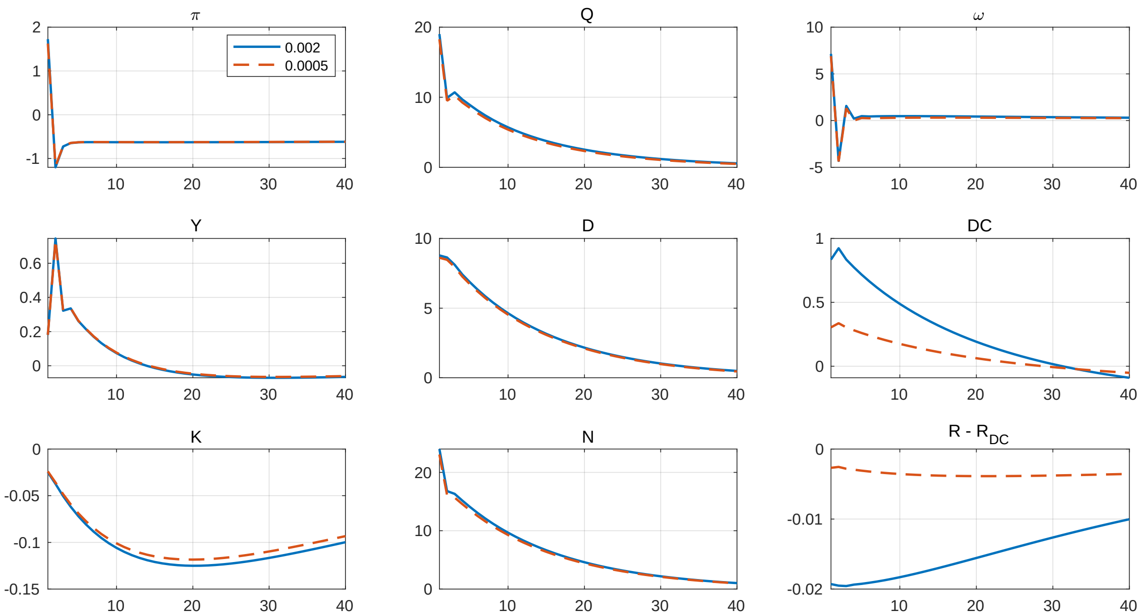


Figure 19: IRFs to a 1bp cost-push shock (low and high $\varkappa^{DC}$)

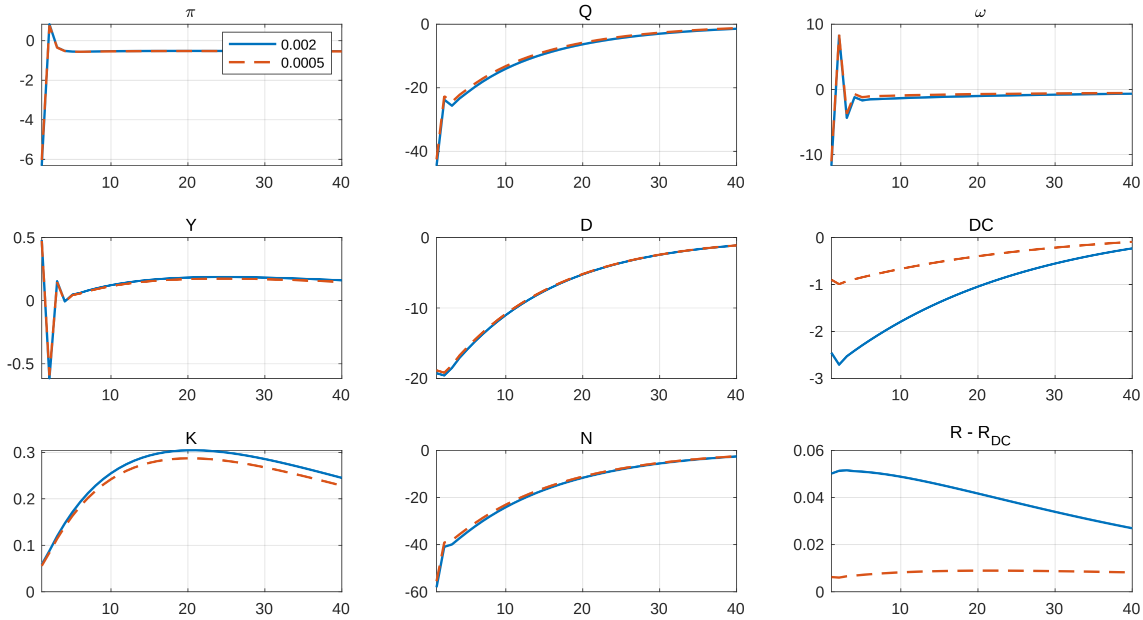


Figure 20: Relative welfare comparisons (% change; $\varkappa^{DC} = 0.002$)

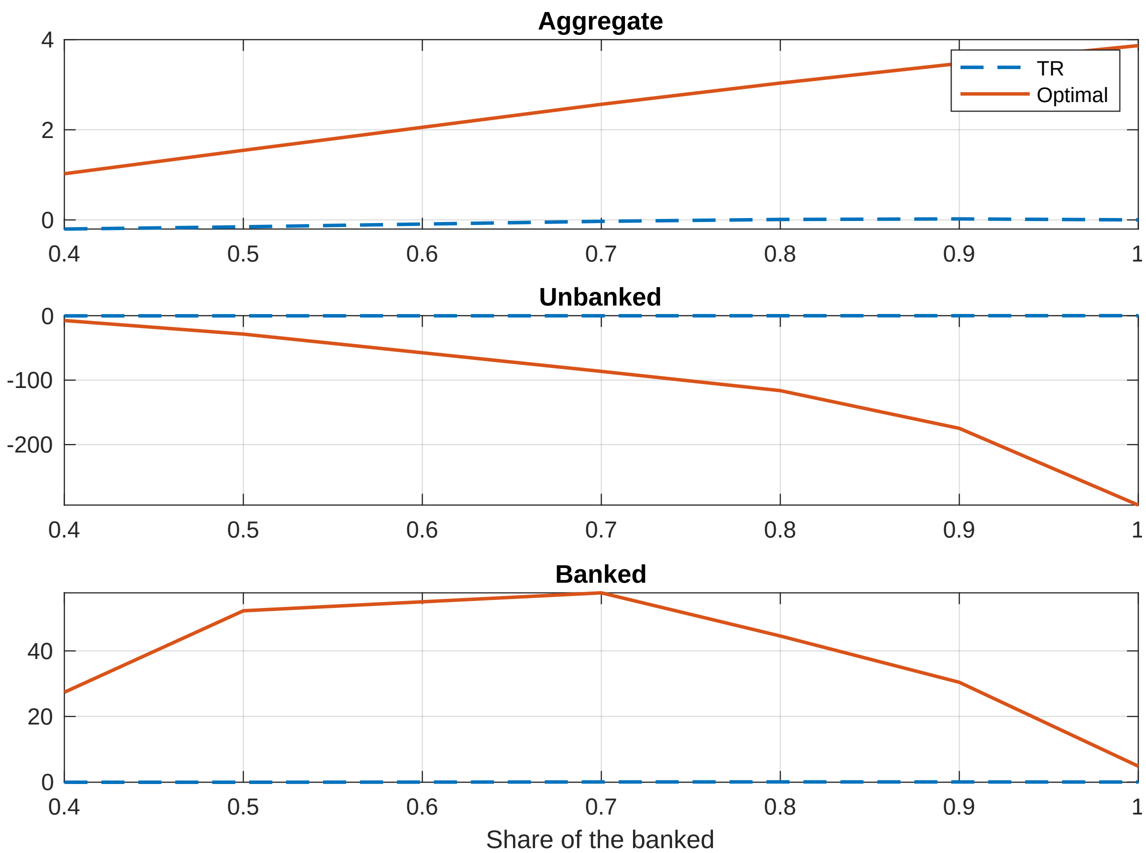


Figure 21: Relative welfare comparisons (% change; $\varkappa^{DC} = 0.0005$)

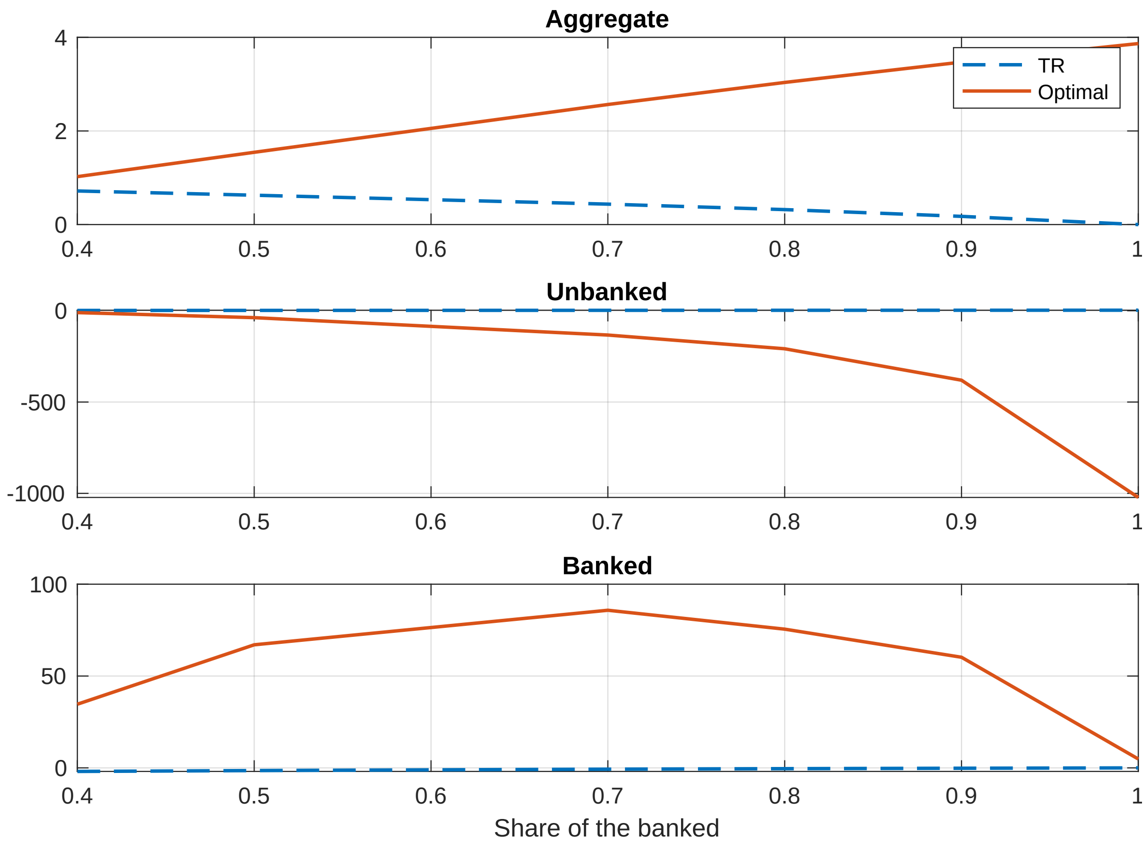


Figure 22: Welfare decomposition, 1% ann. TFP shock ($\varkappa^{DC} = 0.002$)

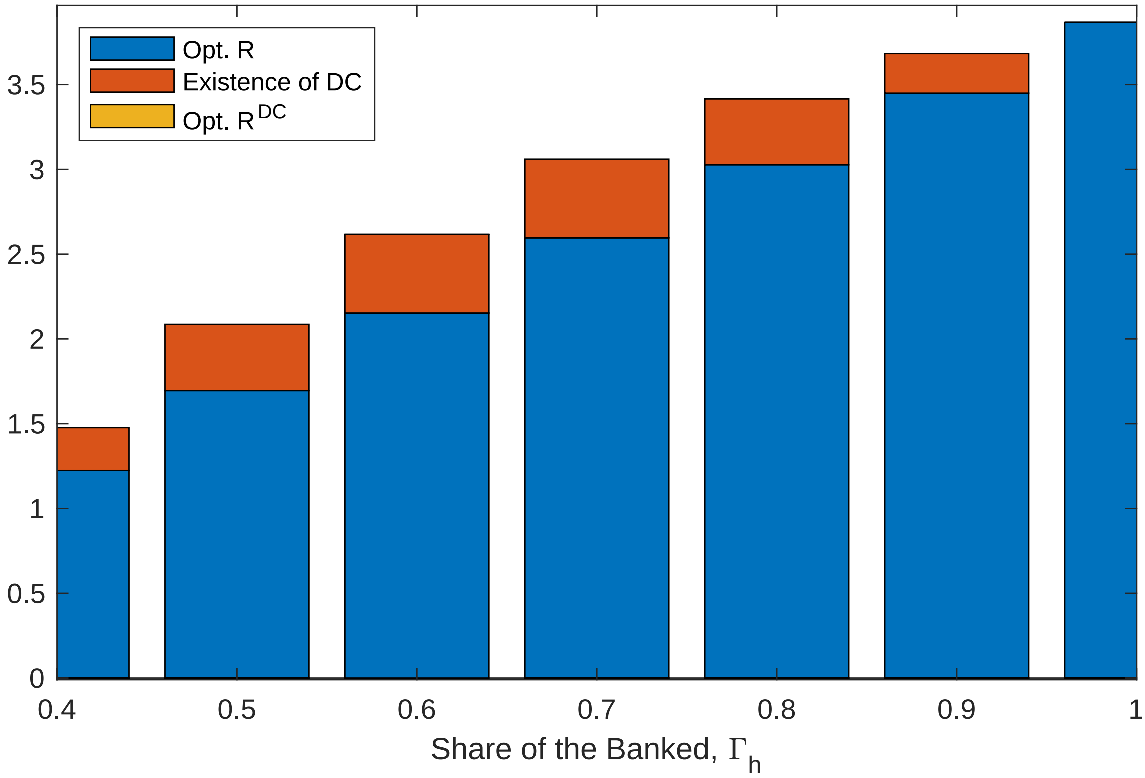
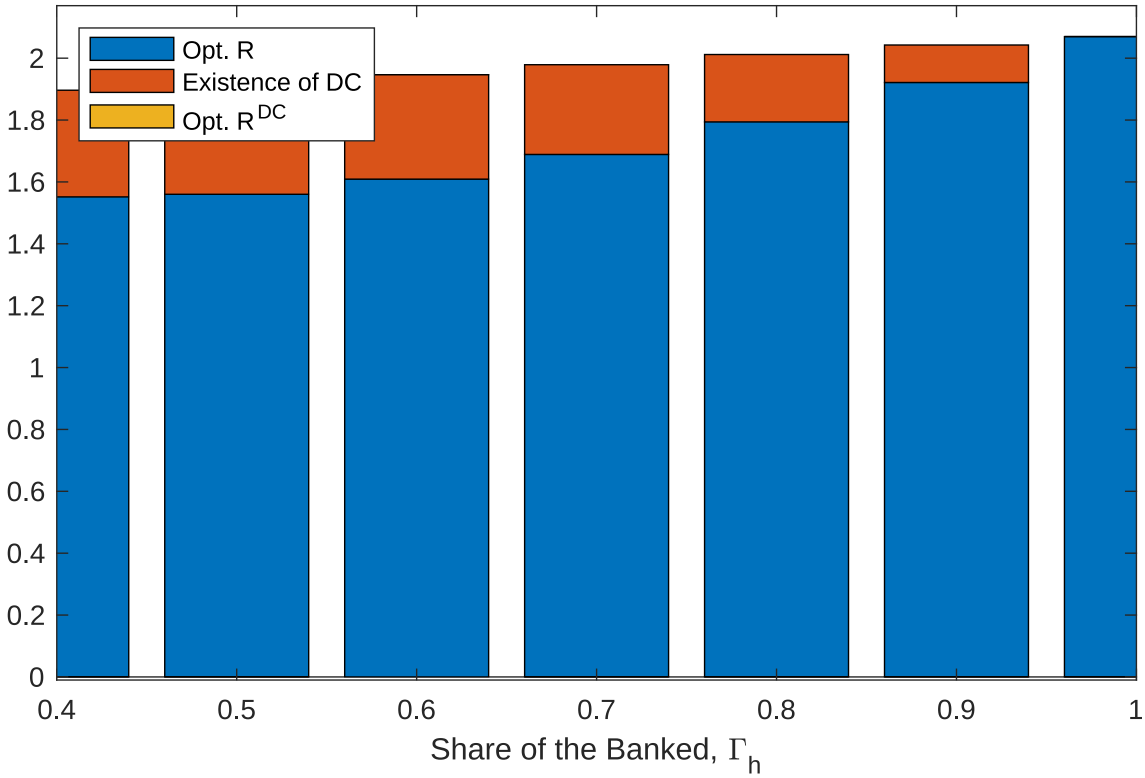


Figure 23: Welfare decomposition, 1% ann. cost-push shock ($\varkappa^{DC} = 0.002$)



A.2.9 Welfare and Optimal Policy

Given the period utility function of the type j household,

$$U_t^j = \ln\left(C_t^j - \zeta_0^j \frac{(L_t^j)^{1+\zeta}}{1+\zeta}\right),$$

the second-order Taylor expansion of U_t^j about the deterministic steady state $(\bar{C}^j, \bar{L}^j)$ is:

$$\begin{aligned} U_t^j - \bar{U} \simeq\ & \bar{U}_C^j \bar{C}^j \left(\frac{C_t^j - \bar{C}^j}{\bar{C}^j}\right) + \bar{U}_L^j \bar{L}^j \left(\frac{L_t^j - \bar{L}^j}{\bar{L}^j}\right) \\ & + \frac{1}{2}\bar{U}_{CC}^j (\bar{C}^j)^2 \left(\frac{C_t^j - \bar{C}^j}{\bar{C}^j}\right)^2 + \frac{1}{2}\bar{U}_{LL}^j (\bar{L}^j)^2 \left(\frac{L_t^j - \bar{L}^j}{\bar{L}^j}\right)^2 + \\ & + \bar{U}_{CL}^j \bar{C}^j \bar{L}^j \left(\frac{C_t^j - \bar{C}^j}{\bar{C}^j}\right)\left(\frac{L_t^j - \bar{L}^j}{\bar{L}^j}\right), \end{aligned}$$

where we ignore terms independent of policy, and where:

$$\begin{aligned} \bar{U}_C^j &= \frac{1}{\bar{C}^j - \zeta_0^j \frac{(\bar{L}^j)^{1+\zeta}}{1+\zeta}}, \\ \bar{U}_L^j &= -\frac{\zeta_0^j}{\bar{C}^j - \zeta_0^j \frac{(\bar{L}^j)^{1+\zeta}}{1+\zeta}} (\bar{L}^j)^\zeta, \\ \bar{U}_{CC}^j &= -\frac{1}{\left(\bar{C}^j - \zeta_0^j \frac{(\bar{L}^j)^{1+\zeta}}{1+1\zeta}\right)^2}, \\ \bar{U}_{LL}^j &= \frac{-\left(\bar{C}^j - \zeta_0^j \frac{(\bar{L}^j)^{1+\zeta}}{1+1\zeta}\right)\zeta_0^j \zeta (\bar{L}^j)^{\zeta-1} - \left(\zeta_0^j (\bar{L}^j)^\zeta\right)^2}{\left(\bar{C}^j - \zeta_0^j \frac{(\bar{L}^j)^{1+\zeta}}{1+1\zeta}\right)^2}, \\ \bar{U}_{CL}^j &= -\frac{\zeta_0^j}{\left(\bar{C}^j - \zeta_0^j \frac{(\bar{L}^j)^{1+\zeta}}{1+\zeta}\right)^2} (\bar{L}^j)^\zeta. \end{aligned}$$