CBDCs, Financial Inclusion, and Monetary Policy

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Abstract

In this paper we study the macroeconomic effects of introducing a retail central bank digital currency (CBDC). Using a two agent framework and endowment economy with banked and unbanked households, we show digital currencies address financial inclusion of the unbanked, by providing a savings vehicle they allow households to smooth consumption. Finally, we study the monetary policy implications in a New Keynesian setting. Welfare gains under strict inflation targeting is higher for a retail indirect CBDC with a primarily unbanked population. An indirect CBDC distributed to households by commercial banks is preferred to a direct CBDC distributed by the central bank. Taken together, our findings suggest a stronger use case for CBDCs in emerging economies with a lower degree of financial inclusion.

Keywords: Central Bank Digital Currency, financial inclusion, inequality, monetary policy, Taylor rules, welfare.

JEL Classifications: TBD

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1 Introduction

Central bank digital currencies (CBDC) are digital tokens, similar to a cryptocurrency, issued by a central bank. Central banks are actively studying the potential adoption of CBDCs, and notable examples include Sweden's E-Krona and China's Digital Currency Electronic Payment. In this emerging literature there is a focus on the macroeconomic effects (Kumhof et al. 2021; Benigno, Schilling, and Uhlig 2022; Ferrari Minesso, Mehl, and Stracca 2022), and implications for banking (Chiu et al. 2019; Skeie 2019; Keister and Sanches 2021; Agur, Ari, and Dell'Ariccia 2022). While CBDCs present obvious advantages – the increased financial inclusion of the unbanked population, improving cross-border payments, and facilitating fiscal transfers – there are still many unresolved issues in their design. For example, Do CBDCs attenuate or amplify monetary policy transmission channels? Is the interest rate adjustable or fixed? Is there a distinction between retail CBDCs which is distributed directly to households by the central bank or indirectly through the commercial banks?

In answering these questions, our paper focuses on CBDC design and in particular the financial inclusion effects of introducing a digital currency. The paper is divided into three parts. First, we review the arguments for and against a retail CBDC using a simple endowment economy with two types of agents. We then extend the model to examine the macroeconomic effects of issuing a digital currency when there is a banking sector and production. This allows us to evaluate the relative benefits of a direct or indirect retail design. Finally we evaluate monetary policy rules in a New Keynesian setup, and determine the magnitude of monetary policy transmission for each CBDC design.

In the first part, we start with a simple two agent endowment economy with a representative banked household (BHH) and unbanked household (UHH). The UHH uses money while the BHH have access to deposits.[1] We introduce a digital currency that can be used by the UHH as an alternative to cash. The central bank can pay an interest rate on this retail CBDC. The primary benefit of this digital currency is that it is a more effective savings vehicle as it relaxes the cash-in-advance (CIA) constraint of the UHH. Welfare for both sets of households improve with a retail CBDC; provided that the net interest rate on the digital currency is positive. However negative rates can lead to potential net welfare losses. This contributes to policy discussions on the role of retail CBDCs in being able to directly charge

1. The BHH and UHH can be thought of as Ricardian and non-Ricardian households, respectively, as is typical in the two-agent New Keynesian literature. See, for example, Debortoli and Galí (2017).

negative rates, giving the monetary authority the ability to tax household money balances. While this may have some merits in periods of low inflation, from a welfare perspective we show that this creates a cost.

A second research question we answer is on CBDC design, and whether a retail CBDC should be direct or indirect.[2] An indirect retail system is one in which financial intermediaries distribute the digital currency, and all households hold digital currency deposits with a retail bank (financial intermediary). A direct retail system is when household types hold digital currency with the central bank, and therefore it is no longer on the financial intermediary's balance sheet. A clear difference between the two regimes is that a direct retail CBDC – though it addresses financial inclusion of the unbanked population – is problematic if banked households substitute away from bank deposits to hold their savings directly with the central bank. This causes financial dis-intermediation of the banking system.

To illustrate this point we extend the baseline model to include production and a financial intermediary that lends to firms that use capital in production. In this setup, we have a trade-off between direct and indirect retail CDBCs. So while the direct CBDC can help the unbanked, it can have negative welfare effects in general equilibrium if a significant fraction of banked households substitute toward the retail CBDC, dis-intermediating banks and reducing capital and production. We evaluate welfare of the retail designs with respect to an economy with no digital currency. We hypothesize that the relative welfare of a indirect CBDC is increasing in the share of BHHs.

Our results show that an indirect CBDC design is the "first-best" design, strictly dominating the direct CBDC design in our baseline calibration. Conducting a sensitivity analysis with respect to the share of the BHH, we find the net benefits to retail CBDCs are higher for an economy primarily populated by UHH. AWe also find an indirect CBDC design is strictly preferred to a direct CBDC for both BHH and UHH.

In the third part of the paper, we add monetary policy and endogenous labor supply to the model with monopolistic pricing of firms. Does a CBDC attenuate or amplify monetary policy transmission? We hypothesize that transmission of monetary policy depends crucially on the CBDC design. For an indirect retail system, monetary policy still transmits through to digital currency deposits. If the CBDC rate can be set independently to a Taylor rule, then it will attenuate monetary policy. Conversely a CBDC rate that follows deposit rates may amplify

2. Some useful references are: https://voxeu.org/article/cbdc-architectures-financial-system-and-central-bank-future and https://voxeu.org/article/central-bank-digital-currencies-drivers-approaches-and-technologies.

transmission. Next, we evaluate welfare criteria for each CBDC design with respect to productivity. Our results suggest that monetary policy transmission is stronger for the indirect retail design. Comparing all regimes under strict inflation targeting, an indirect CBDC design has the highest welfare in an economy populated primarily by UHH. The greatest use case for retail CBDCs therefore lies in emerging markets with low levels of financial inclusion.

The remainder of the paper is structured as follows. In Section 1.1 we summarize the contributions of our paper to related literature. In Section 2 we outline the baseline endowment economy, and examine the welfare implications of introducing a CBDC. In Section 3 we extend our framework to include a financial intermediary and production. Using this model we examine the welfare implications of the direct and indirect CBDC design. In Section 4 we outline a two-agent New Keynesian (TANK) framework, evaluating different rules for the digital currency and optimal inflation targeting across different CBDC designs. Section 5 concludes the paper.

1.1 Related Literature

Our work relates to an emerging literature on the macroeconomic implications of CBDCs (Fernández-Villaverde et al. 2021; Andolfatto 2021; Benigno, Schilling, and Uhlig 2022; Chiu et al. 2019; Keister and Sanches 2021; Benigno 2019; George, Xie, and Alba 2020; Skeie 2019; Ikeda 2020; Kumhof et al. 2021; Cong and Mayer 2021; Agur, Ari, and Dell'Ariccia 2022). The CBDC literature primarily focus on macroeconomic implications. For example, the domestic effects are documented in Kumhof et al. (2021). Skeie (2019) studies an equilibrium in which the cryptocurrency is susceptible to bank runs. The financial intermediation properties of CBDCs have been studied in Keister and Sanches (2021), which determines conditions in which the private sector is dis-inter-mediated with CBDC leading to welfare losses. Chiu et al. (2019) study the role of CBDCs when banks have market power, and show the introduction of CBDCs can lead to increased competition among banks, an increase in deposit rates and lending raising welfare. Our paper is in studying the benefits of CBDCs in a two agent framework. By studying households that do not have access to a financial asset, we focus on the financial inclusion benefits of a retail CBDC.

On the open economy front, Benigno, Schilling, and Uhlig (2022) model a two country framework in which a global stablecoin, like that proposed by Facebook's Libra/Diem, is traded freely between both countries. They determine an equilibrium result of synchronization of interest rates across the two countries in which users are indifferent between holding the global cryptocurrency and the domestic currency. Ferrari Minesso, Mehl, and Stracca (2022) setup a two country model with the CBDC issued by the home country. They find productivity spillovers are amplified in the presence of a CBDC, and it reduces the effectiveness of the foreign country's monetary policy. Cong and Mayer (2021) model the political economy of currency competition with countries choosing between adopting a CBDC and a private cryptocurrency. They show that emerging market countries with weak fundamentals can derive net welfare benefits from cryptocurrency adoption as an alternative to adopting a CBDC or the US dollar. The novelty of our framework in this literature is to include an additional set of households (the unbanked) that do not have access to domestic banking channels. Critically, the unbanked only have access to digital currency as a medium of exchange and savings vehicle. Within this literature we are the first paper to evaluate the welfare benefits of the direct and indirect retail CBDC designs.[3]

The third part of the paper focuses on monetary policy transmission and interest rate rules. Ikeda (2020) models a two-country economy in which goods are priced in foreign currency. Domestic monetary policy transmission is weakened when prices are denominated in a foreign currency. The channel of monetary policy transmission in Ikeda (2020) is expenditure switching; in our paper we offer an alternative channel through having digital currency deposits. Crucially, whether the system is retail or indirect retail matters for monetary policy transmission to bank balance sheets.

2 Simple baseline monetary model

Below we introduce a simple two agent endowment economy. The model comprises two types of households: the banked (BHH) and the unbanked (UHH) with superscripts h and u, respectively. The population is normalized to unity, with the two types of households occupying the continuum $[0, 1]$.

2.1 Banked and unbanked households

BHHs are proportion Γ of the population. They have access to a one-period risk-free savings asset, D_t, which pays a gross nominal rate of interest, R_t. Conversely, the unbanked, of proportion $1 - \Gamma$, do not have access to the safe savings asset, thus their only way of savings is to hold real money balances, M_t.

3. The taxonomy of direct and indirect retail CBDC designs is introduced in Auer and Böhme 2020. They provide many aspects of CBDC design, including architecture (whether it is a direct or indirect claim on the central bank), whether it uses a distributed ledger technology (DLT), account or token based or wholesale or retail. In this paper we focus solely on the architecture of CBDCs.

The infinite horizon problem for the representative BHH is:

$$\max_{\{C^h_{t+s}, D_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{(C^h_{t+s})^{1-\sigma}}{1-\sigma} \right], \tag{1}$$

subject to the period budget constraint (in real terms):

$$C^h_t + D_t = T^h_t + \frac{R_{t-1}}{\pi_t} D_{t-1}, \tag{2}$$

where C^i_t, $i \in \{h, u\}$, is consumption, T^i_t are lump sum transfers, and π_t is gross inflation.[4]

The analogous problem for the representative UHH is:

$$\max_{\{C^u_{t+s}, M_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{(C^u_{t+s})^{1-\sigma}}{1-\sigma} \right], \tag{3}$$

subject to:

$$C^u_t + M_t + \chi^M_t = T^u_t + \frac{M_{t-1}}{\pi_t}, \tag{4}$$

and a CIA constraint:

$$C^u_t \leq \frac{M_{t-1}}{\pi_t}. \tag{5}$$

Note that the UHH pays a quadratic adjustment cost, χ^M_t, to adjust its real money holdings:

$$\chi^M_t = \frac{\phi_M}{2} \left(M_t - \bar{M} \right)^2.$$

We also define ω_t as being an "inequality measure", defined as:

$$\omega_t = 1 - \frac{C^u_t}{C^h_t}, \tag{6}$$

with higher (lower) values of ω_t showing an increase (decrease) in consumption inequality between the BHH and UHH in period t.

As this is a simple monetary model, we assume that there exists a monetary authority which oversees real money balances. We assume the following law of motion for real money balances:

$$M_t = \frac{M_{t-1}}{\pi_t}. \tag{7}$$

4. Gross inflation, π_t, is defined as $\pi_t = P_t / P_{t-1}$, where P_t is the price level.

Finally, endowments are set exogenously and follow a stationary AR(1) process:

$$\ln T_t^i = \rho_T \ln T_{t-1}^i + \varepsilon_t^T, \tag{8}$$

where ε_t^T is exogenous disturbance to both endowments with variance σ_T^2.

For a complete list of equilibrium conditions, please refer to the Appendix A.1.

2.2 Banked and unbanked households with CBDC holdings

In this subsection, we build upon the simple endowment economy presented in Section 2, adding CBDCs which are accessibly by both types of agents.

BHHs maximize their present value discounted stream of utility:

$$\max_{\{C_{t+s}^h, D_{t+s}, DC_{t+s}^h\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \left[\beta^s \left(\frac{(C_{t+s}^h)^{1-\sigma}}{1-\sigma} + \eta_{DC}^h \ln(DC_{t+s}^h) \right) \right], \tag{9}$$

where η_{DC}^h is a scaling parameter and DC_t^h is CBDC holdings. We stipulate that DC_t^h enters the BHH utility function as the CBDC presents the household with non-pecuniary benefits, such as, for example, being able to exchange with financial transactions with the UHH.[5]

The budget constraint of the BHH in real terms is:

$$C_t^h + D_t + DC_t^h = T_t^h + \frac{R_{t-1} D_{t-1} + R_{t-1}^{DC} DC_{t-1}}{\pi_t}. \tag{10}$$

As before, the UHH does not have access to the risk-free saving vehicle D_t, and they continue to face money balances adjustment costs and a CIA constraint. Their optimization problem is otherwise identical as before.

$$\max_{\{C_{t+s}^u, M_{t+s}, DC_{t+s}^u\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \left[\beta^s \left(\frac{(C_{t+s}^u)^{1-\sigma}}{1-\sigma} \right) \right], \tag{11}$$

subject to:

$$C_t^u + M_t + DC_t^u + \chi_t^M = T_t^u + \frac{M_{t-1}}{\pi_t} + \frac{R_{t-1}^{DC} DC_{t-1}^u}{\pi_t}, \tag{12}$$

and the CIA constraint,

$$C_t^u \leq \frac{1}{\gamma} \left(\frac{M_{t-1}}{\pi_t} \right)^{\gamma} (DC_t^u)^{1-\gamma}. \tag{13}$$

5. Additionally, the introduction of DC^h into the utility function of the BHH allows us to determine the optimal level of DC^h holdings.

The policy maker now is able to set the interest rate on digital currency holdings, R_t^{DC}. She uses two types of policy rules that determine the rate of return on digital currency: a constant return rule and a constant spread rule, which are defined as:

$$R_t^{DC} = \begin{cases} R^{\bar{D}C} & \text{const. rate} \qquad (14a) \\ R - \bar{\Delta}^{DC} & \text{const. spread} \qquad (14b) \end{cases}$$

2.3 Welfare comparisons

In order to make welfare comparions across the different regimes and CBDC designs, we define period welfare as having the following functional form:

$$\mathbb{W}_t^i = U(C_t^i) + \beta \mathbb{E}_t \mathbb{W}_{t+1}^i, \tag{15}$$

where $U(\cdot)$ is the period utility function for the representative agent. Note that in order to make accurate welfare comparisons, we use $U(\cdot)$ without CBDC holdings for the BHH.[6]

Additionally, for a description of the parameterization used in our analyses, please refer to Table 2 in Section 4.3.1.

2.3.1 Volatility of endowment

In Figure 1 we plot the ergodic mean[7] of welfare for the BHH, UHH, and the aggregate household[8] against increasing values of the aggregate endowment shock, ε_t^T. We show that allowing the households to have access to an additional instrument of payment and saving is associated with a welfare improvement. Being banked is associated with higher welfare, as it allows the smoothing of consumption through the Euler equation for deposits.

To further illustrate the difference between banked and unbanked households, we observe in Figure 1 that banked households are approximately 2.5% better-off than unbanked households for an endowment shock of 10%. The main difference is that the former have access to deposits, an efficient saving vehicle that allows banked households to smooth their consumption in response to the endowment shock. Thus,

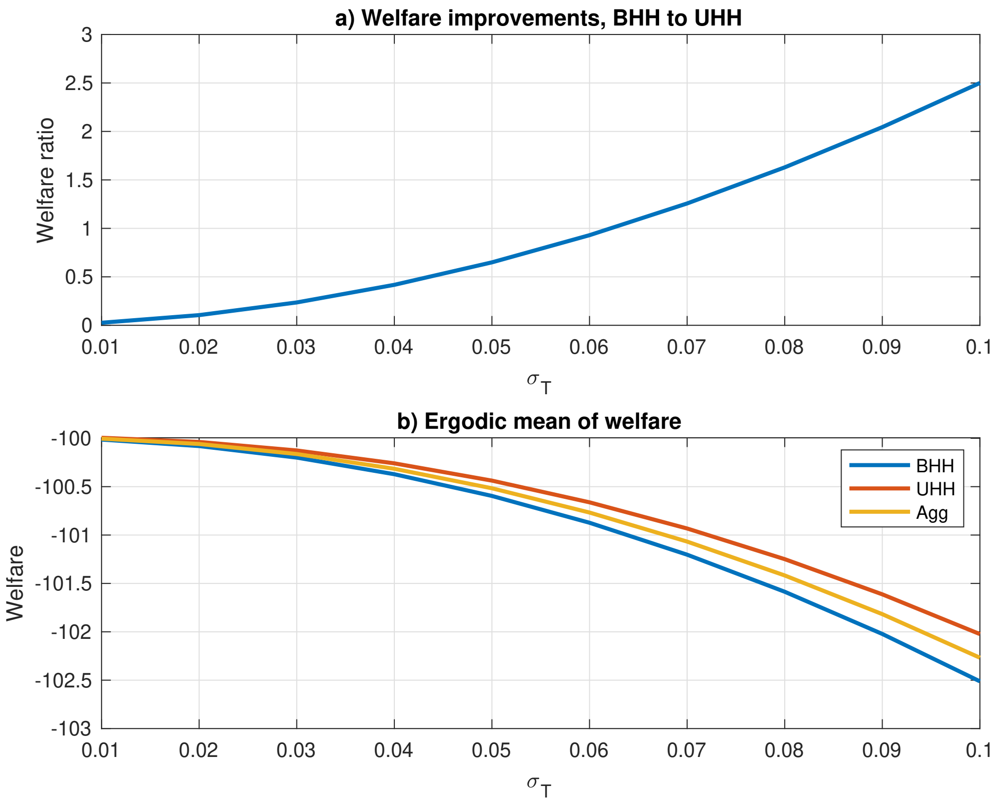
6. Not making this adjustment would distort welfare comparisons, artificially increasing or decreasing welfare due to the holdings of DC for the BHH.

7. To clarify, we take a second-order approximation about the deterministic steady state, subject the economy to our specified shocks, and then simulate the model for 2,000 periods to obtain the mean value of the variables of interest. The ergodic mean is sometimes referred to as the "stochastic steady state".

8. We adjust the population proportion of the representative BHH and UHH when constructing aggregate variables.

providing the unbanked with additional savings vehicle can be welfare improving.

Figure 1: Welfare effects of aggregate endowment shock



2.3.2 Constant rate and constant spread rules

We conduct a welfare exercise across the two regimes with respect to policy rules governing the CBDC rate, R^{DC}, in Figure 2. We use two policy rules for R_t^{DC}, a constant rate and constant spread rule, as shown in Equations (14a) and (14b). In our welfare simulations, we note that the steady state value of R_t^{DC} is bounded by:[9]

$$R^{DC} < 1 + \frac{1-\beta}{\beta + \eta_{DC}^{h}} \tag{16}$$

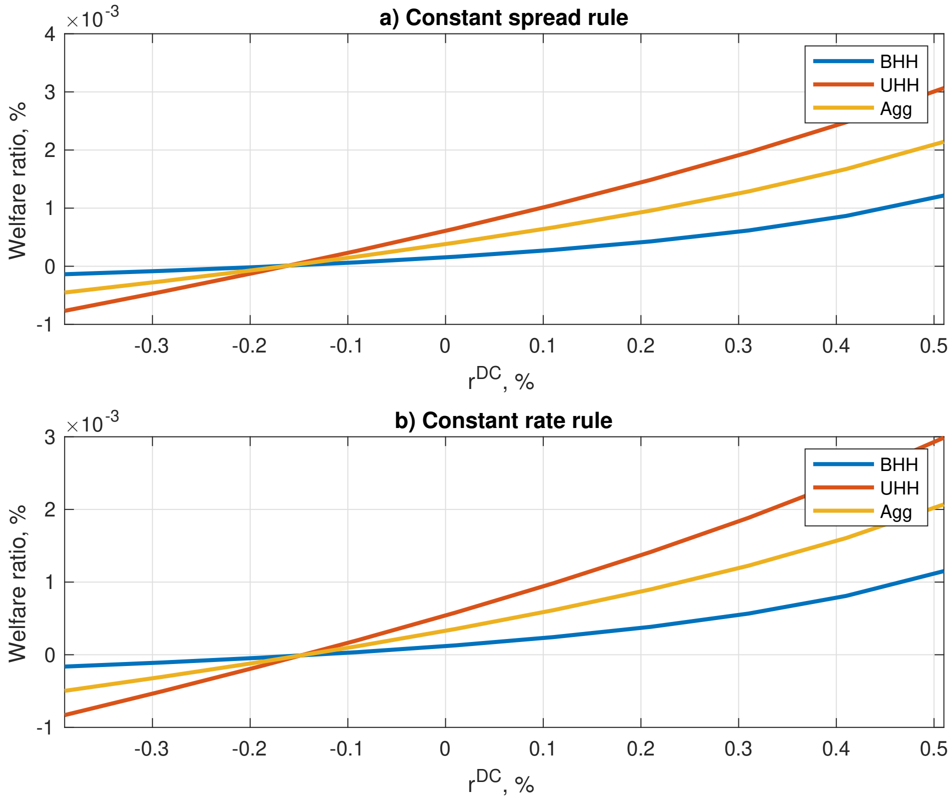
This relationship guarantees the existence of a steady state in the model and thus enforces a lower bound on the rate and the spread of DC. CBDC rates offer a convenience yield relative to deposit rates as there are non-pecuniary motives for holding CBDC in our model. In particular, CBDCs are a hybrid between money and deposits; like money they are legal tender and provide liquidity services, and

9. For full details please refer to the Appendix A.2.4.

like deposits they offer a rate of return.

In Figure 2 we plot the relative welfare gains under digital currency for both the constant spread and fixed CBDC rate rule. Introducing digital currency is welfare improving under both interest rate rules. It is worth mentioning, however, that the constant spread rule in equation (14b) is slightly less welfare improving. This simple setup illustrates that adopting digital currency is welfare improving as it provides an additional savings instruments for the UHH and allows them to smooth consumption through the Euler equation for digital currency holdings. Introducing digital currency is, however, welfare improving only when it is remunerated with a positive rate of return, thus rendering digital currency holdings superior as a savings vehicle to real money balances. We refer to this as a savings channel and document its effects in our models with a banking sector and monetary policy.

Figure 2: Welfare effects of introducing digital currency



Note: r^{DC} is the net nominal interest rate on digital currencies, DC.

3 Monetary model with banking sector

In this section, we extend the model presented in Section 2 in two ways: i) the introduction of a banking sector accompanied with credit frictions; and ii) a supply side of the economy due to physical capital and production. We adopt the setup of Gertler and Karadi (2011), introducing a third type of agent – bankers – which allows us to maintain a representative setup of the household sector. In this setup, banked households hold claims on deposits – denominated in both fiat currency and digital currency – which are held at banks, and they may also directly invest in firms by purchasing equity holdings. Unbanked households are still limited to money holdings and digital currencies; the latter of which are also deposited into the banking sector. Banks then convert deposits into credit, facilitating loans to firms who acquire capital for the means of production, as in Gertler and Kiyotaki (2010, 2015).

We adopt this setup to study welfare implications of different digital currency regimes. The baseline specification that we detail in this section represents a indirect retail CBDC design. By indirect retail regime, we mean that digital currency is stored with banks and, thus, appears on their balance sheets, as described above. We also look into a direct retail CBDC regime, when households store their digital currency with a central bank and, thus, their balances do not appear on commercial banks' balance sheets. We compare welfare implications of direct and indirect CBDC designs to the setup where households do not have access to digital currency.

3.1 Production

The supply side of the economy is simple. Final goods are produced by perfectly competitive firms that use labor and capital to produce their output. They also have access to bank loans, and conditional on being able to take out loan, they do not face any financial frictions. These firms pay back the crediting banks in full via profits. Meanwhile, capital goods are produced by perfectly competitive firms, which are owned by the collective household.

3.1.1 Final good firms

Let there be a continuum of competitive firms which produce final goods, Y_t, from aggregate labor, L_t, and capital, K_t, according to a constant returns to scale (CRTS) production technology:

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{17}$$

where A_t is total factor productivity (TFP) which follows a stationary AR(1) process, and α is the capital share of output. As is standard, the marginal products of output with respect to available capital and aggregate labor supply determine the real rental rate of capital, z_t^k, and the real wage, w_t, respectively:

$$z_t^k = \alpha \frac{Y_t}{K_{t-1}}, \tag{18}$$

$$w_t = (1-\alpha)\frac{Y_t}{L_t}. \tag{19}$$

3.1.2 Capital good firms

We assume that capital goods are produced by perfectly competitive firms, and that the aggregate capital stock grows according to the following law of motion:

$$K_t = I_t + (1-\delta)K_{t-1}, \tag{20}$$

where I_t is investment and $\delta \in (0,1)$ is the depreciation rate.

The objective of the capital good producing firm is to choose I_t to maximize revenue, $Q_t I_t$. Thus, the representative capital good producing firm's objective function is:

$$\max_{I_t} Q_t I_t - I_t - \Phi\left(\frac{I_t}{\bar{I}}\right) I_t,$$

where $\Phi(\cdot)$ are investment adjustment costs as in Christiano, Eichenbaum, and Evans (2005), and are defined as:

$$\Phi\left(\frac{I_t}{\bar{I}}\right) = \frac{\kappa_I}{2}\left(\frac{I_t}{\bar{I}} - 1\right)^2,$$

with $\Phi(1) = \Phi'(1) = 0$ and $\Phi''(\cdot) > 0$. The investment adjustment cost parameter, $\kappa_I = \Phi''(1)$ is chosen so that the price elasticity of investment is consistent with instrumental variable estimates in Eberly (1997).

Differentiating the objective function with respect to I_t gives the FOC:

$$Q_t = 1 + \Phi\left(\frac{I_t}{\bar{I}}\right) + \left(\frac{I_t}{\bar{I}}\right)\Phi'\left(\frac{I_t}{\bar{I}}\right). \tag{21}$$

3.2 Households and workers

The representative household now contains a continuum of individuals, normalized to 1, each of which are of type $i \in \{b, h, u\}$. The setup follows Murakami and Viswanath-Natraj (2021). Bankers ($i = b$) and BHH workers share a perfect insurance scheme, such that they each consume the same amount of real output. However, UHH workers are not part of this insurance scheme, and so their consumption volumes are different from bankers and workers. As before, we define Γ as a proportion of BHH and bankers, hence BHH and bankers are indexed on the continuum $[0, \Gamma]$, whereas the unbanked are indexed on $[\Gamma, 1]$. Much of the setup in Section 2.2 carries over to this section. In what follows, we highlight the key differences introduced in this section.

We endogenize labor supply decisions on the part of households, and so the BHH maximize the present value discounted sum of utility, based on the following utility function:

$$U(C_t^h, DC_t^h, L_t^h) = \frac{(C_t^h)^{1-\sigma}}{1-\sigma} - \zeta_0 \frac{(L_t^h)^{1+\zeta}}{1+\zeta} + \eta_{DC}^h \ln(DC_t^h), \tag{22}$$

subject to their budget constraint:

$$\begin{aligned} &C_t^h + D_t + Q_t K_t^h + DC_t^h + \chi_t^h \\ &\qquad = w_t L_t^h + \Pi_t + (z_t^k + (1-\delta)Q_t)K_{t-1}^h + \frac{R_{t-1}D_{t-1} + R_{t-1}^{DC} DC_{t-1}^h}{\pi_t}, \end{aligned} \tag{23}$$

where w_t are real wages, $L_t^i, i \in \{h, u\}$, is labor supply, ζ is the inverse-Frisch elasticity of labor supply, ζ_0 is a relative labor supply parameter, K_t^h are equity holdings in firms by the BHH, χ_t^h are the costs of equity acquisitions incurred by the BHH, Q_t is the price of equity/capital, and Π_t are distribution of profits due to the ownership of banks and firms. We also note that $\Lambda_{t,t+1}^h$ is the BHH stochastic discount factor (SDF):

$$\Lambda_{t,t+1}^h \equiv \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^h}{C_t^h} \right)^{-\sigma} \right]. \tag{24}$$

For a complete list of FOCs for the BHH problem see Appendix A.3.1.

One distinction between the BHH and bankers purchasing equity in firms is the assumption that the BHH pays an efficiency cost when it adjusts its equity holdings. We assume the following functional form for χ_t^h:

$$\chi_t^h = \frac{\varkappa^h}{2} \left(\frac{K_t^h}{K_t} \right)^2 \Gamma K_t. \tag{25}$$

Meanwhile, the UHH maximizes the present discounted sum of per-period utilities given by

$$U(C_t^u, L_t^u) = \frac{(C_t^u)^{1-\sigma}}{1-\sigma} - \zeta_0 \frac{(L_t^u)^{1+\zeta}}{1+\zeta}, \tag{26}$$

subject to its budget constraint,

$$C_t^u + M_t + DC_t^u + \chi_t^M = w_t L_t^u + \frac{M_{t-1} + R_{t-1}^{DC} DC_{t-1}^u}{\pi_t}, \tag{27}$$

and the CIA constraint, (13).

3.3 Bankers and the finance sector

Among the population of bankers, each j-th banker owns and operates her own bank. A banker will facilitate financial services between households and firms by providing loans to firms in the form of equity, k_t^b, funded by domestic deposits, d_t, and digital currencies deposits, dc_t, and her own net worth, n_t. However, financial frictions may limit the ability of the banker to raise deposits from households.

To this end, each banker seeks to accumulate retained earnings to funds its investments. To maintain model tractability, in each period, bankers have a fixed probability of moving in and out of the financial sector. Let σ_b denote the probability that a banker remains as a banker in the following period, with complementary probability $1 - \sigma_b$ that she retires. This implies an expected franchise life of an individual bank of $\frac{1}{1-\sigma_b}$. Furthermore, the number of bankers exiting the financial market is matched by the number of new bankers entering.

New bankers start up their franchise with fraction γ_b of total assets of the banked households. Upon retirement, a banker will exit with her net worth, bringing the balance back to the household in the form of a dividend. Therefore, a banker will seek to maximize her franchise value, $\mathbb{V}_t^b$, which is the expected present discount value of future dividends:

$$\mathbb{V}_t^b = \mathbb{E}_t \left[\sum_{s=1}^{\infty} \Lambda_{t,t+s}^h \sigma_b^{s-1} (1 - \sigma_b) n_{t+s} \right], \tag{28}$$

where n_{t+s} is the net worth of the bank when the banker retires at date $t + s$ with probability $\sigma_b^{s-1}(1 - \sigma_b)$. Thus, a banker will choose quantities k_t^b, d_t, and dc_t to maximize expression (28).[10]

10. Note that we make the simplifying assumption that each individual banker exogenously accepts digital currency deposits, dc_t, directly in proportion to the population of bankers and total digital currency holdings. In other words, in aggregate, the total sum of individual digital currency deposits at each j-th bank, $dc_t(j)$, is equal to aggregate digital currency deposits, DC_t:

$$\sum_{j=1}^{\infty} dc_t(j) = DC_t.$$

A financial friction inline with Gertler and Kiyotaki (2010) is used to limit the banker's ability to raise funds, whereby the banker faces a moral hazard problem: the banker can either abscond with the funds she has raised from depositors, or the banker can operate honestly and pay out her obligations. Absconding is costly, however, and so the banker can only divert a fraction, $\Theta(\cdot)$, of assets she has accumulated:

$$\Theta(x_t) = \frac{\theta_0^b}{\exp\left(\theta^b x_t\right)}, \tag{29}$$

where we assume that $\{\theta_0^b, \theta^b\} > 0$, and we define x_t as a banker's digital currency deposit leverage ratio:

$$x_t = \frac{dc_t}{Q_t k_t^b}, \tag{30}$$

Thus, following Gertler and Kiyotaki (2010), we assume that as the banker raises a greater proportion of her funds from digital currency deposits, she can only abscond a smaller proportion of her assets. This assumption is supported by the potential for CBDCs to make payments more secure and a reduction in fraud, with more oversight from regulators and the central bank on all transactions of digital currency.[11]

The caveat to absconding, in addition to only being able to take a fraction of assets away, is that it takes time – i.e. it take a full period for the banker to abscond. Thus, the banker must decide to abscond in period t, in addition to announcing what value of d_t she will choose, prior to realizing next period's rental rate of capital. If a banker chooses to abscond in period t, its creditors will force the bank to shutdown in period $t+1$, causing the banker's franchise value to become zero.

Therefore, the banker will choose to abscond in period t if and only if the return to absconding is greater than the franchise value of the bank at the end of period t, $\mathbb{V}_t^b$. It is assumed that the depositors act rationally, and that no rational depositor will supply funds to the bank if she clearly has an incentive to abscond.[12] In other

11. Refer to a Bank of England report for more details on the benefits of digitial currencies, https://www.ukfinance.org.uk/system/files/CBDC-report-FINAL.pdf

12. Consider a simple Gertler and Kiyotaki (2010) setup absent of inflation. Recall that the banker seeks to maximize profits and that it will choose to abscond if and only if:

$$\underbrace{R^k(d+n) - Rd}_{\text{Profit from operating honestly}} \quad < \quad \underbrace{\Theta R^k(d+n).}_{\text{Absconding payoff}}$$

If the banker wants to abscond, she will set her demand for deposits such that the above inequality holds, or,

$$R > \frac{(1-\Theta)R^k(d+n)}{d}.$$

In other words, if a banker signaled that she intended to default, then the return that the worker would receive from depositing with other banks would be greater than the return they would earn by depositing with the absconding banker. Therefore, an absconding banker would receive no words, the bankers face the following incentive constraint:

$$\mathbb{V}_t^b \geq \Theta(x_t) Q_t k_t^b, \tag{31}$$

where we assume that the banker will not abscond in the case of the constraint holding with equality.

3.3.1 Bank balance sheet

Table 1 represents the balance sheet of a typical banker, and so we can write the following balance sheet constraint that the banker faces:

$$\left(1 + \frac{\varkappa^b}{2} x_t^2\right) Q_t k_t^b = d_t + dc_t + n_t, \tag{32}$$

where we assume that the banker faces an efficiency cost from taking in digital currency deposits:

$$\chi_t^b = \frac{\varkappa^b}{2} x_t^2 Q_t k_t^b. \tag{33}$$

Assets	Liabilities + Equity
Loans $Q_t k_t^b$	Deposits d_t
Management costs χ_t^b	Digital currency deposits dc_t
	Net worth n_t

Table 1: Bank balance sheet

Additionally, we can write the flow of funds constraint for a banker as

$$n_t = [z_t^k + (1-\delta) Q_t] k_{t-1}^b - \frac{R_{t-1}}{\pi_t} d_{t-1} - \frac{R_{t-1}^{DC}}{\pi_t} dc_{t-1}, \tag{34}$$

noting that for the case of a new banker, the net worth is the startup fund given by the household:

$$n_t = \gamma_b [z_t^k + (1-\delta) Q_t] k_{t-1}$$

3.3.2 Rewriting the banker's problem

With the constraints of the banker established, we can proceed to write the banker's problem as:

$$\max_{k_t, d_t} \mathbb{V}_t^b = \mathbb{E}_t \left[\Lambda_{t,t+1}^h \left\{(1-\sigma_b) n_{t+1} + \sigma_b \mathbb{V}_{t+1}^b\right\}\right],$$

deposits, and so an optimizing banker would not choose to abscond.

subject to the incentive constraint (31) and the balance sheet constraint (32).

Since $\mathbb{V}_t^b$ is the franchise value of the bank, which we can interpret as a "market value", we can divide $\mathbb{V}_t^b$ by the bank's net worth to obtain a Tobin's Q ratio for the bank denoted by ψ_t:

$$\psi_t \equiv \frac{\mathbb{V}_t^b}{n_t} = \mathbb{E}_t \left[\Lambda_{t,t+1}^h (1 - \sigma_b + \sigma_b \psi_{t+1}) \frac{n_{t+1}}{n_t} \right]. \tag{35}$$

We then define ϕ_t as the maximum feasible asset to net worth ratio, or, rather, the leverage ratio of a bank:

$$\phi_t = \frac{Q_t k_t^b}{n_t}. \tag{36}$$

Additionally, if we define $\Omega_{t,t+1}$ as the stochastic discount factor of the banker, μ_t as the excess return on capital over fiat currency deposits, μ_t^{DC} as the cost advantage of digital currency deposits over fiat currency deposits, and υ_t as the marginal cost of deposits, we can write the banker's problem as the following:

$$\psi_t = \max_{\phi_t} \left\{ \mu_t \phi_t + \left(1 - \frac{\varkappa^b}{2} x_t^2 \phi_t\right) \upsilon_t + \mu_t^{DC} x_t \phi_t \right\}, \tag{37}$$

subject to

$$\psi_t \geq \Theta(x_t) \phi_t. \tag{38}$$

Solving this problem yields:

$$\phi_t = \frac{\upsilon_t}{\Theta(x_t) - \mu_t - \mu_t^{DC} x_t + \frac{\varkappa^b}{2} x_t^2 \upsilon_t}, \tag{39}$$

where:

$$\mu_t = \mathbb{E}_t \left[\Omega_{t,t+1} \left\{ \frac{z_{t+1}^k + (1-\delta) Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}} \right\} \right], \tag{40}$$

$$\mu_t^{DC} = \mathbb{E}_t \left[\Omega_{t,t+1} \left\{ \frac{R_t}{\pi_{t+1}} - \frac{R_t^{DC}}{\pi_{t+1}} \right\} \right], \tag{41}$$

$$\upsilon_t = \mathbb{E}_t \left[\Omega_{t,t+1} \frac{R_t}{\pi_{t+1}} \right], \tag{42}$$

$$\Omega_{t,t+1} = \Lambda_{t,t+1}^h (1 - \sigma_b + \sigma_b \psi_{t+1}). \tag{43}$$

For the complete solution of the banker, please refer to Appendix A.3.2 and A.3.3.

3.4 Market equilibrium

Aggregate consumption, labor supply, and digital currency holdings by the BHH and UHH are given as:

$$C_t = \Gamma C_t^h + (1 - \Gamma)C_t^u, \tag{44}$$

$$\bar{L} = \Gamma \bar{L}^h + (1 - \Gamma)\bar{L}^u, \tag{45}$$

$$DC_t = \Gamma DC_t^h + (1 - \Gamma)DC_t^u. \tag{46}$$

The aggregate resource constraint of the economy is:

$$Y_t = C_t + \left[1 + \Phi\left(\frac{I_t}{\bar{I}}\right)\right] I_t + \Gamma\chi_t^h + (1 - \Gamma)\chi_t^M + \Gamma\chi_t^b, \tag{47}$$

with aggregate capital being given by:

$$K_t = \Gamma(K_t^h + K_t^b). \tag{48}$$

Aggregate net worth of the bank is given by:

$$\begin{aligned} N_t = \sigma_b &\left[(z_t^k + (1 - \delta)Q_t)K_{t-1}^b - \frac{R_{t-1}}{\pi_t}D_{t-1} - \frac{R_{t-1}^{DC}}{\pi_t}\frac{DC_{t-1}}{\Gamma}\right] \\ &+ \gamma_b(z_t^k + (1 - \delta)Q_t)\frac{K_{t-1}}{\Gamma}, \end{aligned} \tag{49}$$

and the aggregate balance sheet of the bank is given by the following equations:

$$Q_t K_t^b = \phi_t N_t, \tag{50}$$

$$Q_t K_t^b \left(1 + \frac{\varkappa^b}{2}x_t^2\right) = D_t + \frac{DC_t}{\Gamma} + N_t, \tag{51}$$

$$x_t = \frac{DC_t}{Q_t \Gamma K_t^b}. \tag{52}$$

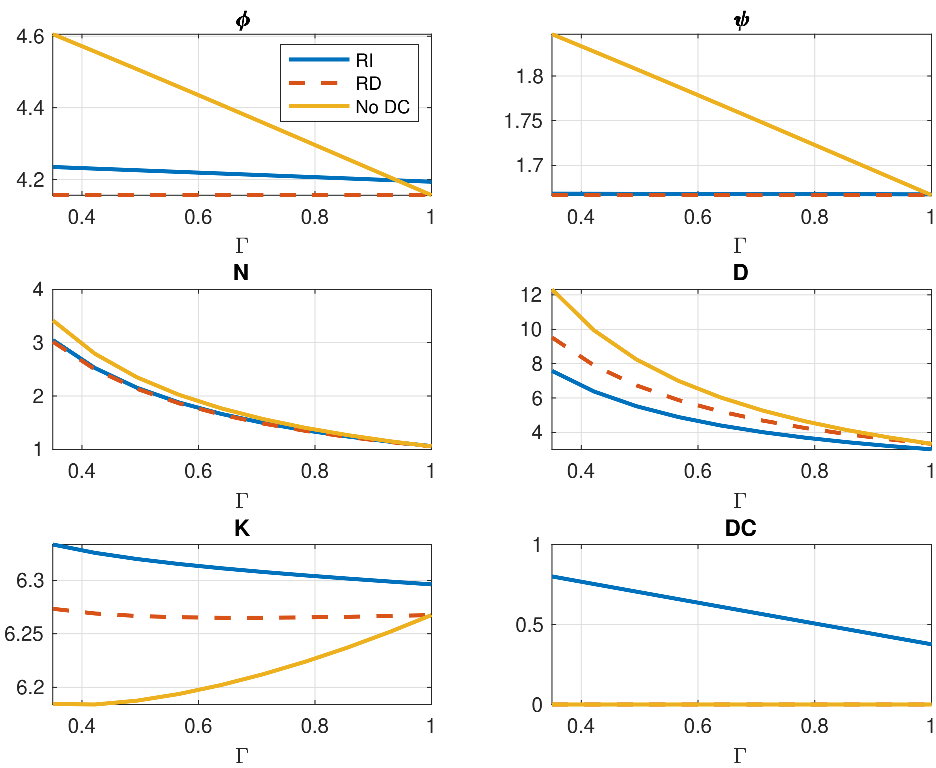
3.5 Quantitative experiments

We explore the efficiency of bank intermediation in this section. The ability of banks to efficiently provide capital to producers explains the welfare difference between the direct and indirect CBDC designs. A direct retail distribution means that digital currency is distributed directly by the central bank to households. Therefore this dis-intermediates the bank. The retail digital currency regime implies that all the digital currency holdings are stored with a central bank, but not with a commercial bank. The regime under which digital currency is not present in the economy is straightforward. It implies that BHH save only in the form of deposit claims D_t and hold equity K_t^h, while the UHH only operate real money balances M_t as their savings vehicle.

3.5.1 Steady state

To illustrate the differences between the direct and indirect retail CBDCs, we start with the steady state implications in Figure 3. We observe that in steady state the banker has higher leverage, ϕ, and higher net worth, N in an indirect CBDC design. Mathematically, we can see that the optimal leverage of the banker is increasing in the share of digital currency deposits x in Equation (30). That directly translates into bankers' ability to finance higher steady state values of capital. We also see that deposits, D_t, that banks use to finance equity, are lower under indirect retail than under both direct retail and no CBDC regimes, but are compensated through digital currency holdings on the bank balance sheet. The differences in steady state leverage, net worth and capital is important in understanding the welfare effects on households.

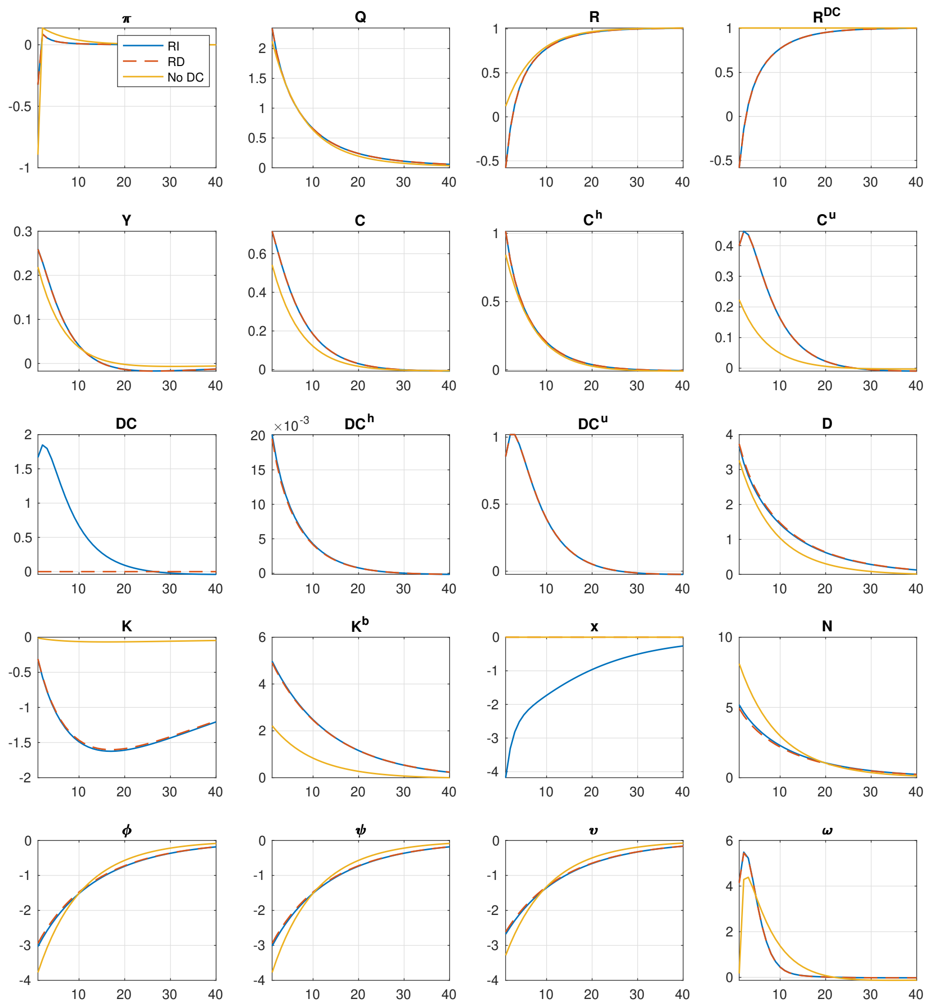
Figure 3: Steady state implications of CBDC regimes



3.5.2 Impulse responses

Figure 4 presents impulse response functions with respect to a one-standard deviation (1%) TFP shock. We compare an indirect retail CBDC to the economy with no digital currency. The output are stronger in the regime with digital currency, and is strongest for the indirect retail CBDC. This is because of the general equilibrium effects of digital currency deposits on the bank balance sheet increasing net worth, leverage and capital supplied in production. Stronger consumption responses in the indirect retail regime is due to wealth effects of an increase in production.

Figure 4: IRFs to 1% TFP shock, indirect retail and no CBDC regimes



3.5.3 Welfare comparisons

We evaluate welfare implications of two digital currency regimes: a direct and indirect retail CBDC relative to a model with no digital currency in Figure 5.[13] We calculate welfare with respect to a 1% TFP shock with persistence $\rho_A = 0.9$, changing the proportion of BHH, Γ, in the economy under the constant spread rule for digital currency in Equation (14b). We observe that direct and indirect digital currency regimes dominate the no CBDC regime for UHH.

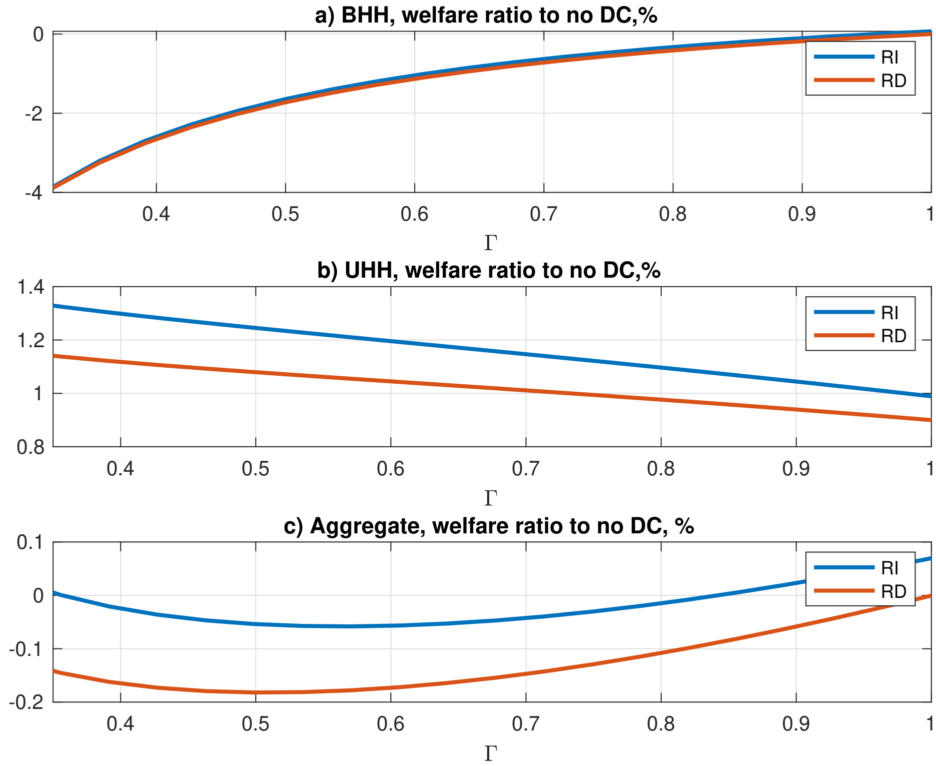
We attribute the positive welfare benefits of a retail CBDC to the savings channel. The retail CBDC allows UHH to use a savings vehicle that remunerates them and, thus, allows them to increase consumption relative to the economy with no digital currency. This explains the welfare improvements associated with adopting digital currency. As the savings channel mainly concerns the unbanked, its welfare improving effects dissipate when the share of the unbanked decreases. The difference between the direct and indirect retail CBDC depends on the financial intermediation channel. An indirect CBDC is welfare enhancing as digital currency deposits increase the bank's net worth and leverage. This translates to increased lending to firms, higher capital and production, and increased consumption through general equilibrium effects. Therefore the indirect retail CBDC is a preferred system in our setup.

The BHH, however, achieve net welfare losses with CBDCs. Our hypothesis is that the CBDCs introduces an amplification of the consumption response and bank balance sheets to productivity shocks. This amplification leads to excess volatility relative to the no-CBDC regime, and is higher when Γ is low and the economy is primarily populated by unbanked households. On the other hand, BHH do not gain from either the savings or financial intermediation channel relative to the no-CBDC regime, as they already have an efficient savings vehicle.

Turning to the aggregate household, we find there are net welfare benefits relative to the no-CBDC regime when Γ is very low, so the economy is primarily unbanked, or when Γ is very high and approaches 1. When Γ is low this corresponds to the case when the savings and financial intermediation channels dominate for UHH and the gains from financial inclusion are strongest. When Γ is very high, there are still net benefits to the unbanked, and the welfare losses for BHH due to excess volatility from the introduction of the retail CBDC is minimal.

13. Note that we once again suppress the role of DC in the BHH utility function in order to make fairer welfare comparisons.

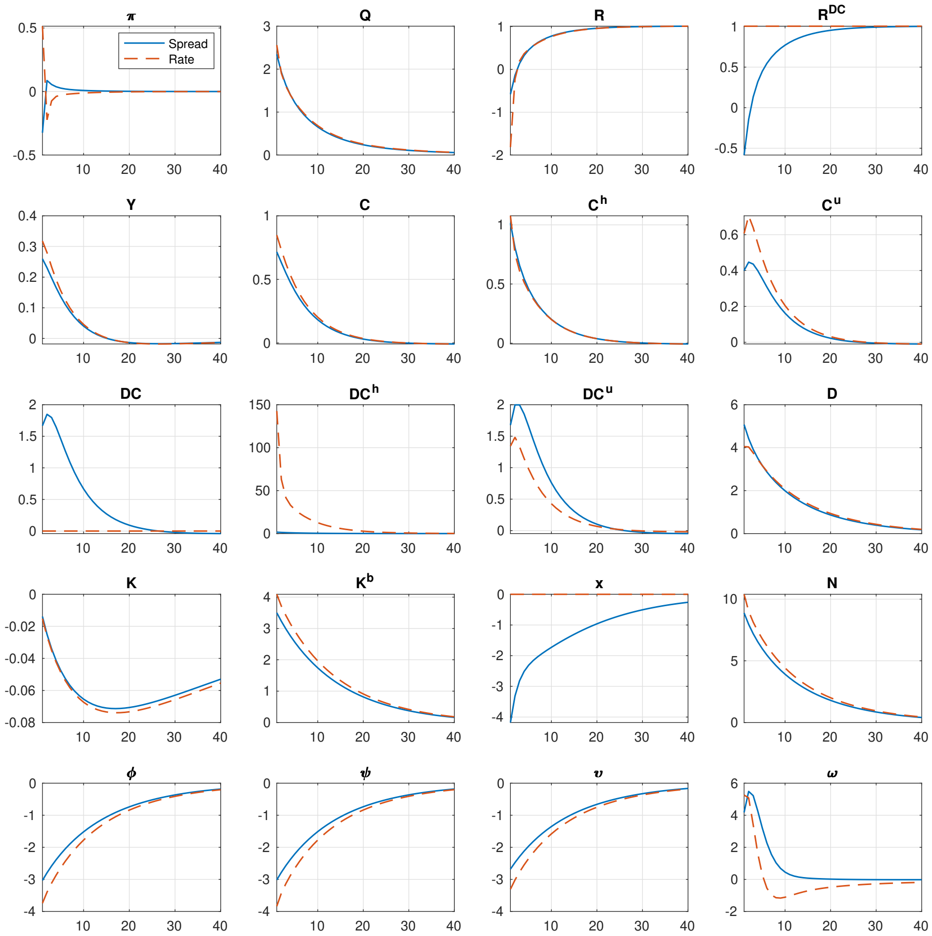
Figure 5: Welfare implications of different DC regimes, constant spread rule



3.5.4 CBDC rules: constant versus spread

Figure 6 presents impulse response functions with respect to a one-standard deviation TFP shock. For a indirect retail CBDC economy, we compare a fixed CBDC rate rule and a constant spread rule. The output effects are similar under both policy rules, however the main differences are in the response of deposits. Commercial bank and digital currency deposits are higher for the fixed rate rule. Deposit rates fall more with a fixed rate rule, and consumption is slightly more responsive.

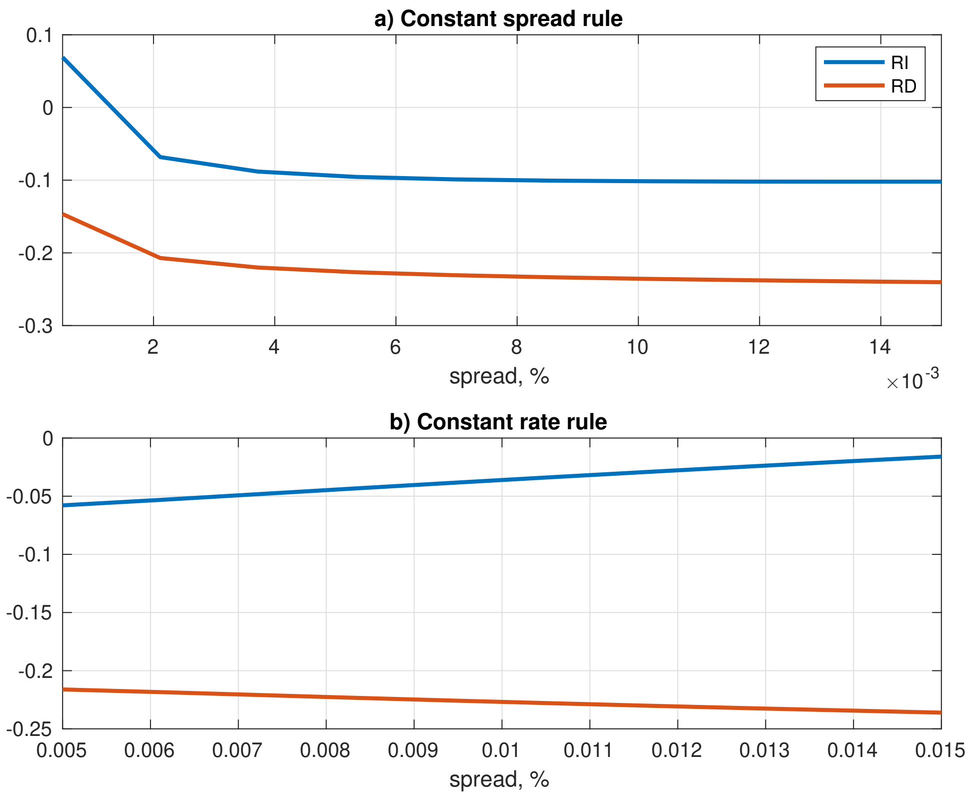
Figure 6: IRFs to 1% TFP shock, indirect retail CBDC design



The welfare comparison across the two rules is provided in Figure 7. We assume $\Gamma = 0.5$. Comparing the constant spread and fixed rate rules, we note that the constant rate rule does not provide welfare improvements. One can notice that while under constant spread rule the welfare improvements are positive for small values of the spread, the constant rate rule is never welfare improving. Negative digital currency rates are better than the positive ones in terms of welfare.

These findings can be explained by two factors. Firstly, setting a constant rate on digital currency is akin to a passive response to exogenous disturbances. In other words, the constant rate rule amplifies the TFP shock and, thus, decreases ergodic mean of welfare. The constant rate rule implies inefficient movements of digital currency holdings of BHH; as the deposit rate decreases, with digital currency rate staying constant, the BHH are incentivized to hold inefficiently higher amounts of digital currency. This explains welfare losses in both digital currencies. Secondly, the lower the rate on digital currency, the less the households are incentivized to hold it, attenuating the channel related to the inefficient digital currency accumulation. This explains why a constant negative rate implies higher welfare than a positive rate for the constant rate rule in Figure 7. As the constant rate rule is not welfare improving for any feasible values of R^{DC}, we abstract from it further in the paper and only explore the constant spread rule when extending the model to include monetary policy.

Figure 7: Welfare implications of different CBDC designs



Note: RI denotes retail indirect and RD denotes retail direct.

4 Two-agent New Keynesian model with digital currency holdings

In this section, we introduce price stickiness and monopolistic competition (Christiano, Eichenbaum, and Evans 2005; Smets and Wouters 2007; Galí 2015) to build a two-agent New Keynesian (TANK) model. Much of the setup is inherited from

Section 3, and here we highlight key additions to the model.

4.1 Production

4.1.1 Final goods producer

There is a representative competitive final good producing firm which aggregates a continuum of differentiated intermediate inputs according to a Dixit-Stiglitz aggregator:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} \, dj \right)^{\frac{\epsilon}{\epsilon-1}}, \epsilon > 0. \tag{53}$$

So final good firms maximize their profits by selecting how much of each intermediate good to purchase, and so their problem is:

$$\max_{Y_t(j)} P_t Y_t - \int_0^1 P_t Y_t(j) dj.$$

Solving for the FOC for a typical intermediate good j is:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t. \tag{54}$$

The relative demand for intermediate good j is dependent of j's relative price with ϵ, the price elasticity of demand, and is proportional to aggregate output, Y_t.

From Blanchard and Kiyotaki (1987), we can derive a price index for the aggregate economy:

$$P_t Y_t \equiv \int_0^1 P_t(j) Y_t(j) dj.$$

Then, plugging in the demand for good j from (54) we have:

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}.$$

4.1.2 Intermediate goods producers

The continuum of intermediate good producers are normalized to have a mass of unity. A typical intermediate firm j produces output according to a CRTS technology in capital and labor with a common productivity shock:

$$Y_t(j) = A_t K_{t-1}(j)^{\alpha} L_t(j)^{1-\alpha}.$$

The problem for the j-th firm is to minimize costs,

$$\min_{K_{t-1}(j), L_t(j)} z_t^k K_{t-1}(j) + W_t L_t(j),$$

subject to their production constraint:

$$A_t K_{t-1}(j)^\alpha L_t(j)^{1-\alpha} \geq Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t.$$

This yields the minimized unit cost of production:

$$MC_t = \frac{1}{A_t}\left(\frac{z_t^k}{\alpha}\right)^\alpha \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}. \tag{55}$$

The price-setting problem of firm j is set up à la Calvo (1983). Price stickiness arises from the fact that a producer is not able to adjust prices in the next period with probability θ; and is able to with complimentary probability $1-\theta$. The producer maximizes its expected discounted[14] value of profits:

$$\max_{P_t(j)} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \Lambda_{t,t+s}^h \theta^s \left\{(1-\tau)\frac{P_t(j)}{P_{t+s}} Y_{t+s}(j) - MC_{t+s} Y_{t+s}(j)\right\}\right],$$

where τ is a lump-sum subsidy to offset distortions to the steady state arising from monopolistic competition, the producer markup, $\mathcal{M} = \frac{\epsilon}{\epsilon-1} > 1$. From the intermediate firm's pricing problem, we attain expressions for price dispersion, Δ_t:

$$\Delta_t = \theta \pi_t^\epsilon \Delta_{t-1} + (1-\theta)\left(\frac{x_{1,t}}{x_{2,t}}\right)^{-\epsilon}, \tag{56}$$

and inflation dynamics:

$$\pi_t^{1-\epsilon} = \theta + (1-\theta)(\pi_t^\#)^{1-\epsilon}, \tag{57}$$

where we define the reset price inflation, $\pi_t^\#$,

$$\pi_t^\# = \frac{\mathcal{M}}{(1-\tau)} \frac{x_{1,t}}{x_{2,t}} \pi_t, \tag{58}$$

and the auxiliary variables, $x_{1,t}$ and $x_{2,t}$:

$$x_{1,t} = Y_t (C_t^h)^{-\sigma} MC_t + \theta \mathbb{E}_t \Lambda_{t,t+1}^h \pi_{t+1}^\epsilon x_{1,t+1}, \tag{59}$$

$$x_{2,t} = Y_t (C_t^h)^{-\sigma} + \theta \mathbb{E}_t \Lambda_{t,t+1}^h \pi_{t+1}^{\epsilon-1} x_{2,t+1}. \tag{60}$$

14. Note that the firm is owned by the BHH and thus uses the SDF $\Lambda_{t,t+1}^h$.

Finally, using the production function for each intermediate firm, we have the aggregate resource constraint:

$$Y_t = \frac{A_t K_{t-1}^{\alpha} L_t^{1-\alpha}}{\Delta_t} \tag{61}$$

4.1.3 Efficient steady state and resulting targeting rule

As noted above, there is a distortion arising from monopolistic competition in production. We use lump-sum subsidy to offset the distortion. We assume that a policy maker can use lump-sum taxes to pay for a lump sum subsidy τ in steady state. It is worth noting, however, that the policy maker in our setup is not able to use taxes and/our subsidies as policy instruments out of steady state.

From equation (58), we see that the policy maker chooses subsidy such that

$$\frac{\mathcal{M}}{(1-\tau)} = 1 \implies \tau = -\frac{1}{\epsilon - 1}$$

which guarantees a non-distorted steady-state. This is crucial for the optimal simple rules experiments that follow. An efficient level of steady state output implies that the policymaker chooses to target welfare relevant output gap, which is the ratio of realized output Y_t and flexible price output Y_t^f. Hereinafter, we abstract from distorted steady-state and only consider the efficient one.

4.2 Monetary policy

The central bank is assumed to operate an inertial Taylor Rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_R} \left(\pi_t^{\phi_\pi} X_t^{\phi_Y}\right)^{1-\rho_R} \exp(\varepsilon_t^R) \tag{62}$$

where it reacts to inflation and the output gap, X_t, which we define as:

$$X_t = \frac{Y_t}{Y_t^f}, \tag{63}$$

where Y_t^f is the flexible price level of output. Finally, ε_t^R an exogenous and transitory monetary policy shock.

4.3 Quantitative experiments

4.3.1 Parameterisation and steady state values

Table 2: Model parameter values

Parameter	**Value**	**Description**
θ	0.100	Elasticity of leverage wrt foreign borrowing
θ_0	0.401	Severity of bank moral hazard
σ	0.940	Survival probability
γ^b	0.0045	Fraction of total assets inherited by new banks
$\varkappa^b$	0.000	Management cost for DC
β	0.990	Discount rate
σ	2.000	Risk aversion
ζ	0.333	Inverse-Frisch elasticity
ζ_0	7.883	Inverse labour supply capacity
η^h_{DC}	0.001	BHH preference for DC
$\varkappa^h$	0.020	Cost parameter of direct finance
Γ	0.500	Proportion of BHH
γ	0.800	CIA weight on money
ϕ_M	10.000	Money adjustment cost parameter
α	0.333	Capital share of output
δ	0.025	Depreciation rate
ϵ	9.000	Elasticity of demand
κ_I	0.667	Investment adjustment cost
$\bar{A}$	1.000	Steady state TFP
$\bar{i}$	0.010	Steady state net nominal interest rate
$\bar{\pi}$	0.000	Steady state net inflation
θ	0.750	Calvo parameter
τ	0.111	Producer subsidy
$\mathcal{M}$	1.125	Intermediate producer markup
Δ^R	0.005	Spread on deposits and DC
ϕ_π	1.500	Taylor rule inflation coefficient
ϕ_Y	0.500	Taylor rule output coefficient
ρ_A	0.850	AR(1) coefficient for TFP shock
ρ_R	0.900	Taylor rule persistence
σ_A	0.01	Standard deviation of TFP shock
σ_R	0.0025	Standard deviation of MP shock

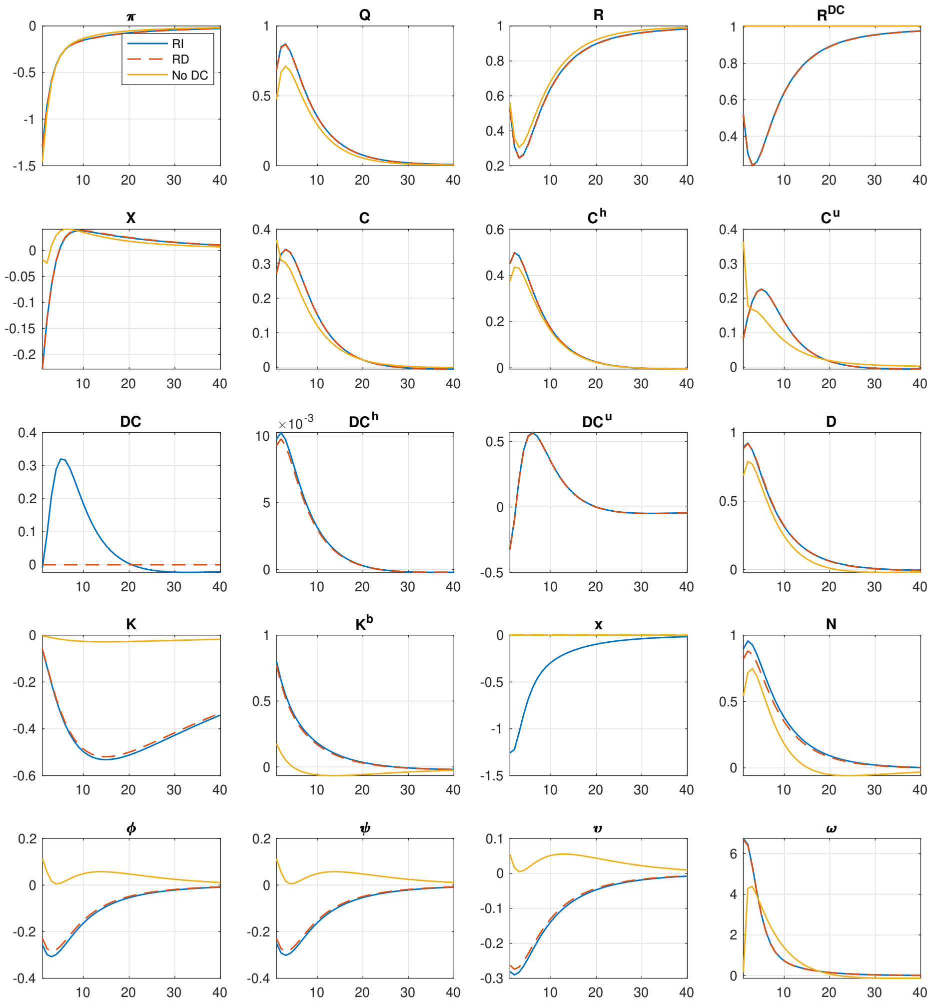
4.3.2 Equilibrium dynamics of the TANK model with CBDCs

We illustrate the dynamics of the model with respect to a TFP shock and a monetary policy shock under a conventional Taylor Rule and the two digital currency rate rules. Figure 8 plots the results in response to a one-standard deviation TFP shock. Relative to the no-CBDC regime, an expansionary productivity shock has stronger effects on consumption, the output gap and bank balance sheets. The intuition is as follows. The DC gives unbanked households a savings vehicle. This enables the UHH to save more following the productivity shock, which leads to an accumulation of digital currency deposits. This enables higher consumption for the unbanked households over a longer period, even though the contemporaneous effect on consumption is stronger for the no-CBDC case. The ability of the bank to raise digital currency deposits increases the net worth of the bank. Asset prices are higher and the bank invests in more capital relative to the regime with no-CBDC. In summary, the introduction of a retail CBDC leads to an amplification of the productivity shock for consumption, the net worth and lending of banks.

Figure 9 presents results in response to a 25 basis point monetary policy shock. Relative to the no-CBDC regime, a contractionary monetary policy shock now transmits to UHH. We see strong pass-through to UHH consumption, with small effects in the no-CBDC regime. The consumption response for UHH translates to stronger transmission to aggregate consumption and the output gap. The differences between the indirect and direct retail CBDC are negligible, and only affect the banking variables, in particular the digital currency deposits now appear on the bank balance sheet. However, it does not materially affect the responses of bank leverage, net worth and lending across the two retail CBDCs.

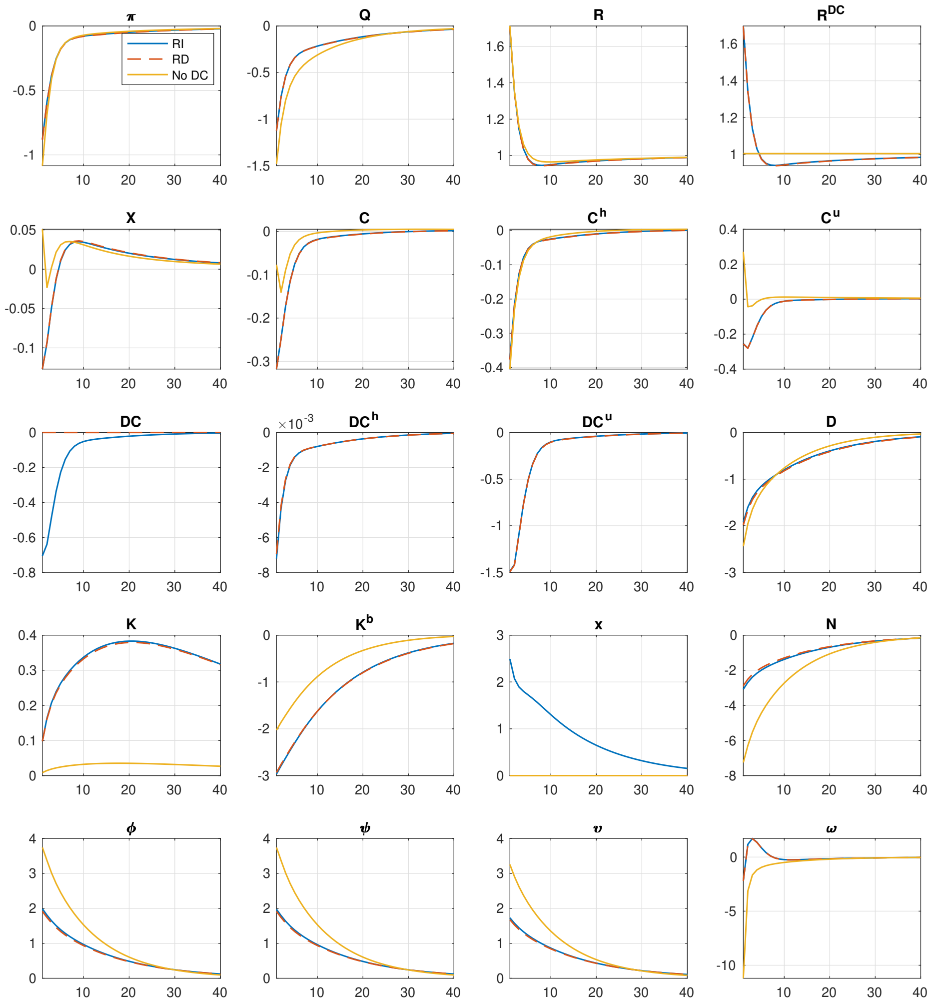
Comparing banking responses of a retail CBDC to the no CBDC case, we find the transmission to banking variables is muted with a retail CBDC. This is intuitive: the monetary shock transmits to bank balance sheets through changing the cost of bank deposits, which in turn affects the leverage and net worth directly in the no CBDC case. In a retail CBDC model, the share of bank deposit share reduces. This reduces the sensitivity of the bank balance sheet to monetary shocks. However, while the effect on bank is attenuated with a retail CBDC, the savings channel of the CBDC means the consumption response of the UHH dominate. In summary, monetary policy transmission to consumption is strengthened, and transmission to bank balance sheets is attenuated with a retail CBDC.

Figure 8: IRFs to 1% TFP shock under constant spread rule



Note: IRFs show percent deviations from steady state values, except for inflation and interest rates which show annualized net rates.

Figure 9: IRFs to 25 b.p. monetary policy shock under constant spread rule



Note: IRFs show percent deviations from steady state values, except for inflation and interest rates which show annualized net rates.

4.3.3 Welfare analysis and optimal policy

We conduct an optimal policy exercise across the three setups (direct retail, indirect retail and no-CBDC) and two digital currency rate policy regimes (constant spread and constant rate rules) under an instrumental monetary policy rule of the form:

$$\pi_t X_t^{\phi_Y} = 1, \tag{64}$$

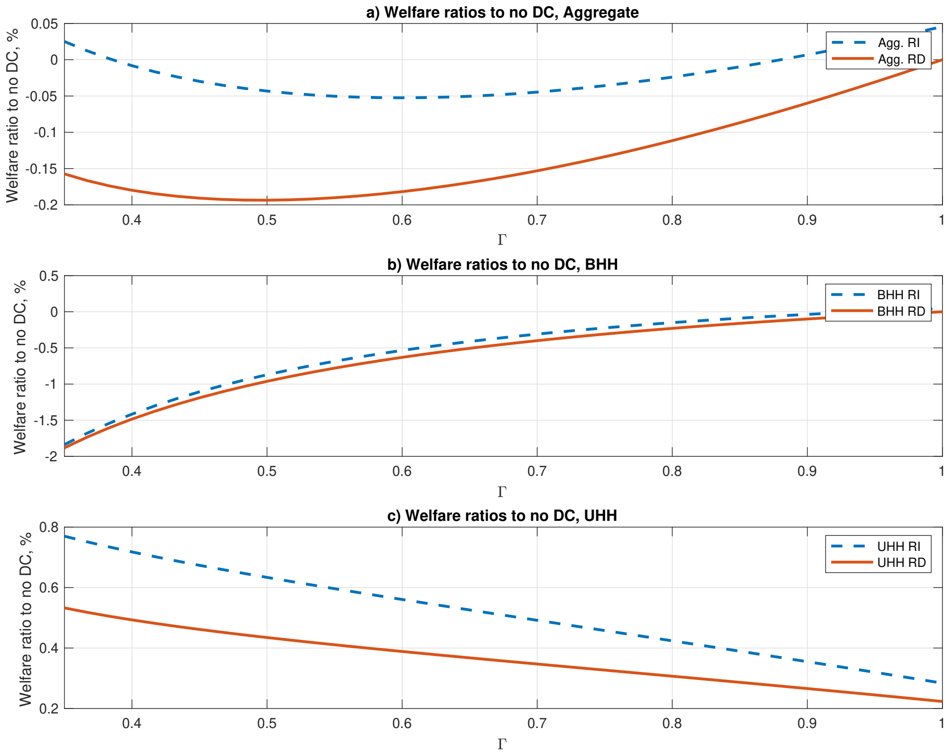
where here ϕ_Y determines the degree to which the policy maker cares about stabilizing the welfare relevant output gap. This rule implies strict inflation targeting if $\phi_Y = 0$, which turns out to be optimal for all setups and digital currency rules.

We evaluate welfare implications of the direct and indirect retail CBDC relative to a model with no digital currency in Figure 10. We calculate welfare with respect to a one-standard deviation TFP shock with persistence $\rho_A = 0.9$ and a one-standard deviation monetary shock of 25 basis points.

Similar to our RBC setup in Section 3, the UHH have lower levels of financial inclusion and have a stronger incentive to adopt a retail CBDC. This reflects the savings channel. As the CBDC offers a rate of remuneration, it is an effective savings vehicle for the unbanked and enables them to achieve welfare gains through smoothing consumption. Comparing the two types of retail CBDC, we note that the indirect retail CBDC dominates. The gains of an indirect retail CBDC are larger when the unbanked population is larger: this is because of the financial intermediation channel. As the unbanked population now hold digital currency deposits through a bank, this increases the bank net worth relative to a direct retail CBDC. The bank is able to lend more to firms, increasing production and consumption through general equilibrium effects.

The BHH, however, achieve net welfare losses with a retail CBDC. Our hypothesis is that a retail CBDC introduces an amplification of the consumption response and bank balance sheets to both productivity and monetary policy shocks. This amplification leads to excess volatility relative to the no-CBDC regime, and is higher when Γ is low and the economy is primarily populated by unbanked households. On the other hand, the BHH does not gain from either the savings or financial intermediation channel relative to the no-CBDC regime, as they already have an efficient savings vehicle. Turning to aggregate household, we find there are net welfare benefits relative to the no-CBDC regime when Γ is very low, so the economy is primarily unbanked, or when Γ is very high and approaches 1. When Γ is low this corresponds to the case when the savings and financial intermediation channels dominate for the UHH and the gains from financial inclusion are strongest. When Γ is very high, there are still net benefits to the unbanked, and the welfare losses for BHH due to excess volatility from the introduction of the retail CBDC is minimal.

Figure 10: Welfare comparison across regimes



5 Conclusion

In this paper we focus on the financial inclusion effects of introducing a digital currency. We address a number of research questions on the welfare implications of a retail CBDC design, such as whether it is a direct claim on a central bank or whether it is facilitated through commercial banks, whether interest rates are adjustable or fixed, and on the strength of monetary policy transmission after digital currency adoption.

In the first part we review the arguments for and against a retail CBDC using a simple endowment economy with two types of agents. Welfare for both sets of households improve with a retail CBDC provided the interest rate on CBDC is positive. However negative rates can lead to potential net welfare losses.

We then extend the model to examine the macroeconomic effects of issuing a digital currency when there is a financial intermediary and production. This allows us to evaluate the relative benefits of a direct or indirect retail design. Our results show that a indirect retail system is the “first-best” design, with the welfare of an indirect retail CBDC strictly dominating the direct retail CBDC regime in our baseline calibration. Conducting a sensitivity analysis with respect to the share of the banked population, we find the net benefit to a retail CBDC is stronger for unbanked households. This is consistent with the savings channel: unbanked households now have access to an asset that remunerates a rate of return and assists in consumption smoothing. Second, the unbanked households strictly prefer an indirect retail CBDC. This is because they gain from a financial intermediation channel through facilitating an increase in the deposits and net worth of the bank.

Finally we evaluate monetary policy rules in a New Keynesian setup, and determine the magnitude of monetary policy transmission for each CBDC design. Our results suggest that the introduction of retail CBDC amplifies monetary policy transmission to consumption. This is because the unbanked households now hold digital currency deposits and are sensitive to the central bank rate. Comparing all regimes under strict inflation targeting, we find an indirect retail design has the highest welfare in an economy populated primarily by unbanked households. Taken together, our findings suggest the greatest use case for retail CBDCs lies in emerging markets with low levels of financial inclusion.

References

Agur, Itai, Anil Ari, and Giovanni Dell'Ariccia. 2022. "Designing central bank digital currencies." *Journal of Monetary Economics* 125:62–79.

Andolfatto, David. 2021. "Assessing the impact of central bank digital currency on private banks." *The Economic Journal* 131 (634): 525–540.

Auer, Raphael, and Rainer Böhme. 2020. "The technology of retail central bank digital currency." *BIS Quarterly Review, March.*

Benigno, Pierpaolo. 2019. "Monetary Policy in a World of Cryptocurrencies." Centre of Economic Policy Research, *discussion papers.*

Benigno, Pierpaolo, Linda M. Schilling, and Harald Uhlig. 2022. "Cryptocurrencies, Currency Competition, and the Impossible Trinity." *Journal of International Economics,* 103601.

Blanchard, Olivier J., and Nobuhiro Kiyotaki. 1987. "Monopolistic Compeitition and the Effects of Aggregate Demand." *American Economic Review* 77 (4): 647–666.

Calvo, Guillermo A. 1983. "Staggered Prices in a Utility-Maximising Framework." *Journal of Monetary Economics* 12 (3): 383–398.

Chiu, Jonathan, Seyed Mohammadreza Davoodalhosseini, Janet Hua Jiang, and Yu Zhu. 2019. "Bank market power and central bank digital currency: Theory and quantitative assessment." *Available at SSRN 3331135.*

Christiano, Lawrence J., Martin S. Eichenbaum, and Charles L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113 (1): 1–45.

Cong, Lin William, and Simon Mayer. 2021. "The Coming Battle of Digital Currencies." *Available at SSRN 3992815,* https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4068564.

Debortoli, Davide, and Jordi Galí. 2017. "Monetary Policy with Heterogeneous Agents: Insights from TANK Models." Department of Economics and Business, Universitat Pompeu Fabra, *Economics Working Papers,* no. 1686.

Eberly, Janice C. 1997. "International Evidence on Investment and Fundamentals." *European Economic Review* 41 (6): 1055–1078.

Fernández-Villaverde, Jesús, Daniel Sanches, Linda Schilling, and Harald Uhlig. 2021. "Central bank digital currency: Central banking for all?" *Review of Economic Dynamics* 41:225–242.

Ferrari Minesso, Massimo, Arnaud Mehl, and Livio Stracca. 2022. "Central Bank Digital Currency in an Open Economy." *Journal of Monetary Economics.*

Galí, Jordi. 2015. *Monetary Policy, Inflation, and the Business Cycle.* 2nd Edition. Princeton University Press.

George, Ammu, Taojun Xie, and Joseph D Alba. 2020. "Central Bank Digital Currency with Adjustable Interest Rate in Small Open Economies." *Available at SSRN 3605918,* https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3605918.

Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics* 58 (1): 17–34.

Gertler, Mark, and Nobuhiro Kiyotaki. 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis." *Handbook of Monetary Economics* 3:547–599.

———. 2015. "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy." *American Economic Review* 105 (7): 2011–2043.

Ikeda, Daisuke. 2020. "Digital Money as a Unit of Account and Monetary Policy in Open Economies." Institute for Monetary and Economic Studies, Bank of Japan.

Keister, Todd, and Daniel R Sanches. 2021. "Should central banks issue digital currency?" *Available at SSRN 3966817.*

Kumhof, Michael, Marco Pinchetti, Phurichai Rungcharoenkitkul, and Andrej Sokol. 2021. "Central Bank Digital Currencies, Exchange Rates and Gross Capital Flows." *ECB working paper series,* no. No 2488, https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp2488~fede33ca65.en.pdf.

Murakami, David, and Ganesh Viswanath-Natraj. 2021. "Cryptocurrencies in Emerging Markets: A Stablecoin Solution?" *SSRN working paper,* https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3949012.

Skeie, David R. 2019. "Digital Currency Runs." *Available at SSRN 3294313,* https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3294313.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review* 97 (3): 586–606.

A Appendix

A.1 Solution to baseline endowment economy model

A.1.1 Full set of equilibrium conditions

Euler equation for banked households:

$$(C_t^h)^{-\sigma} = \beta(C_{t+1}^h)^{-\sigma}\frac{R_t}{\pi_{t+1}} \tag{65}$$

Euler equation for unbanked households:

$$\frac{\beta}{\pi_{t+1}}(C_{t+1}^u)^{-\sigma} = \lambda_t^u(1+\phi_M(M_t-\bar{M})); \tag{66}$$

Marginal utility of consumption for unbanked households:

$$(C_t^u)^{-\sigma} = \lambda_t^u + \mu_t^u; \tag{67}$$

Budget constraint for unbanked households:

$$C_t^u + M_t + \frac{\phi_M}{2}\left(M_t - \bar{M}\right)^2 = T_t^u + \frac{M_{t-1}}{\pi_t}, \tag{68}$$

CIA constraint:

$$C_t^u = \frac{M_{t-1}}{\pi_t} \tag{69}$$

Money supply:

$$M_t = \frac{M_{t-1}}{\pi_t}, \tag{70}$$

Aggregate resource constraint:

$$\Gamma\left[T_t^u - M_t - \frac{\phi_M}{2}(M_t-\bar{M})^2\right] + (1-\Gamma)(T_t^h - C_t^h) = 0; \tag{71}$$

A.1.2 Analytical steady state

$$\bar{R} = \frac{1}{\beta}$$

$$\bar{\pi} = 1$$

$$\bar{C^h} = \bar{T}_h$$

$$\bar{C^u} = \bar{T_u}$$

$$\bar{\lambda}^u = \beta$$

$$\bar{\mu}^u = 1 - \beta$$

$$\bar{M} = 1$$

A.2 Solution to the endowment economy with digital currency holdings

This subsection outlines solution of the endowment economy model.

A.2.1 Banked households

$$\max_{\{C^h_{t+s}, D_{t+s}, DC^h_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \left[\beta^s \left(\frac{(C^h_{t+s})^{1-\sigma}}{1-\sigma} \right) + \eta^h_{DC} \ln(DC^h_{t+s}) \right], \tag{72}$$

where η^0_{DC} is a scaling parameter, DC^h_t is CBDC holdings.
Budget constraint in real terms:

$$C_t + D_t + DC^h_t + \frac{\phi^h_{DC}}{2}(DC^h_t - \bar{DC}^h)^2 = T^h_t + \frac{R_{t-1}D_{t-1}}{\pi_t} + \frac{R^{DC}_{t-1}DC_{t-1}}{\pi_t} \tag{73}$$

This problem yields the following first-order conditions:

$$(C^h_t)^{-\sigma} = \lambda^h_t \tag{74}$$

$$\beta\lambda^h_{t+1}\frac{R_t}{\pi_{t+1}} = \lambda^h_t \tag{75}$$

$$\beta\lambda^h_{t+1}\frac{R^{DC}_t}{\pi_{t+1}} + \eta^0_{DC}(DC^h_t)^{-1} = \lambda^h_t(1 + \phi^h_{DC}(DC^h_t - \bar{DC}^h)) \tag{76}$$

A.2.2 Unbanked households

$$\max_{\{C^u_{t+s}, M_{t+s}, DC^u_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \left[\beta^s \left(\frac{(C^u_{t+s})^{1-\sigma}}{1-\sigma} \right) \right], \tag{77}$$

Budget constraint in real terms:

$$C_t^u + M_t + DC_t^u + \frac{\phi_M}{2}\left(M_t - \bar{M}\right)^2 = T_t^u + \frac{M_{t-1}}{\pi_t} + \frac{R_{t-1}^{DC} DC_{t-1}^u}{\pi_t} \tag{78}$$

CIA constraint:

$$C_t^u \leq \frac{1}{\gamma}\left(\frac{M_{t-1}}{\pi_t}\right)^{\gamma} (DC_t^u)^{1-\gamma}. \tag{79}$$

I redefine the CIA RHS to be:

$$G_t(M_{t-1}, DC_t^u) = \frac{1}{\gamma}\left(\frac{M_{t-1}}{\pi_t}\right)^{\gamma} (DC_t^u)^{1-\gamma}. \tag{80}$$

With derivatives with respect to M_t and DC_t^u equal to

$$G'_{M_t} = \left(\frac{M_{t-1}}{\pi_t}\right)^{\gamma-1} (DC_t^u)^{1-\gamma} \tag{81}$$

$$G'_{DC_t^u} = (1-\gamma)\left(\frac{M_{t-1}}{\pi_t}\right)^{\gamma} (DC_t^u)^{-\gamma} \tag{82}$$

First-order conditions for the problem are:

Marginal utility of consumption

$$(C_t^u)^{-\sigma} = \lambda_t^u + \mu_t^u \tag{83}$$

Euler equation for real money balances

$$\beta(\frac{\lambda_{t+1}^u}{\pi_{t+1}} + \mu_{t+1} G'_{t+1}(M_t)) = \lambda_t^u (1 + \phi_M^u (M_t - \bar{M})) \tag{84}$$

Euler equation for digital currency holdings

$$\beta \frac{\lambda_{t+1}^u R_t^{DC}}{\pi_{t+1}} + \mu_t^u G'_t(DC_t^u) = \lambda_t^u \tag{85}$$

A.2.3 Full set of equilibrium conditions

BHH

Marginal utility of consumption

$$(C_t^h)^{-\sigma} = \lambda_t^h \tag{86}$$

Euler equation for bond holdings

$$\beta \lambda_{t+1}^{h} \frac{R_t}{\pi_{t+1}} = \lambda_t^h \tag{87}$$

Euler equation for digital currency holdings

$$\beta \lambda_{t+1}^{h} \frac{R_t^{DC}}{\pi_{t+1}} + \eta_{DC}^{h} (DC_t^h)^{-1} = \lambda_t^h \tag{88}$$

UHH

Auxiliary variable G_{t+1} derivative wrt M_t

$$G'_{M_t} = \gamma \left(\frac{M_{t-1}}{\pi_t} \right)^{\gamma - 1} (DC_t^u)^{1-\gamma} \tag{89}$$

Auxiliary variable G_t derivative wrt DC_t

$$G'_{DC_t^u} = (1-\gamma) \left(\frac{M_{t-1}}{\pi_t} \right)^{\gamma} (DC_t^u)^{-\gamma} \tag{90}$$

Marginal utility of consumption

$$(C_t^u)^{-\sigma} = \lambda_t^u + \mu_t^u \tag{91}$$

Euler equation for real money balances

$$\beta (\frac{\lambda_{t+1}^u}{\pi_{t+1}} + \mu_{t+1} G'_{t+1}(M_t)) = \lambda_t^u (1 + \phi_M^u (M_t - \bar{M})) \tag{92}$$

Euler equation for digital currency holdings

$$\beta \frac{\lambda_{t+1}^u R_t^{DC}}{\pi_{t+1}} + \mu_t^u G'_t(DC_t^u) = \lambda_t^u \tag{93}$$

AGGREGATION

Digital currency aggregate

$$DC_t = \Gamma DC_t^h + (1-\Gamma) DC_t^u \tag{94}$$

Digital currency supply

$$R_t^{DC} = R^{\bar{D}C} \tag{95}$$

Money supply

$$M_t = \frac{M_{t-1}}{\pi_t}\xi_t^M, \tag{96}$$

Endowment process for Banked

$$ln(T_t^h) = \rho_T ln(T_{t-1}^h) + \epsilon_{T^h} \tag{97}$$

Endowment process for Unbanked

$$ln(T_t^u) = \rho_T ln(T_{t-1}^u) + \epsilon_{T^u} \tag{98}$$

CIA constraint for the Unbanked

$$C_t^u \leq \frac{1}{\gamma}\left(\frac{M_{t-1}}{\pi_t}\right)^{\gamma}(DC_t^u)^{1-\gamma}. \tag{99}$$

Aggregate resource constraint for the economy

$$\begin{aligned}\Gamma(C_t^h + D_t + DC_t^h) + (1-\Gamma)\left[C_t^u + M_t + DC_t^u + \chi_t^M\right] \\ = \Gamma\left(T_t^h + \frac{R_t D_{t-1}}{\pi_t} + \frac{R_t^{DC} DC_{t-1}}{\pi_t}\right) \\ + (1-\Gamma)\left[T_t^u + \frac{M_{t-1}}{\pi_t} + \frac{R_t^{DC} DC_{t-1}^u}{\pi_t}\right]\end{aligned} \tag{100}$$

A.2.4 Analytical steady-state solution

This section provides analytical steady solution and highlights its implications for values of $R^{\bar{D}C}$.

Digital currency holdings, unbanked

$$\bar{D}C^u = \bar{T}^u\left(\left(\frac{\gamma}{1-\gamma}\frac{1-\beta R^{DC}}{1-\beta}\right)^{\gamma} - R^{\bar{D}C} + 1\right)^{-1} \tag{101}$$

Real money holdings

$$\bar{M} = \bar{T}^u\left(\left(\frac{\gamma}{1-\gamma}\frac{1-\beta R^{DC}}{1-\beta}\right)^{\gamma} - R^{\bar{D}C} + 1\right)^{-1}\frac{\gamma}{1-\gamma}\frac{1-\beta R^{DC}}{1-\beta} \tag{102}$$

Consumption, unbanked

$$\bar{C}^u = \bar{T}^u + \bar{T}^u\left(\left(\frac{\gamma}{1-\gamma}\frac{1-\beta R^{DC}}{1-\beta}\right)^{\gamma} - R^{\bar{D}C} + 1\right)^{-1}(R^{\bar{D}C} - 1) \tag{103}$$

Digital currency holdings, banked

$$\bar{DC^h} = \frac{\eta^h_{DC}\bar{T^h}}{(1-\beta R^{DC}) - \eta^0_{DC}(R^{DC}-1)} \tag{104}$$

Consumption, banked

$$\bar{C^h} = \bar{T^h} + \frac{\eta^h_{DC}\bar{T^h}}{(1-\beta R^{DC}) - \eta^h_{DC}(R^{DC}-1)}(R^{DC}-1) \tag{105}$$

Analytical steady-state solution is non-trivial for the general case. We solve the simple endowment economy model with digital currency assuming that $\sigma = 1$ for tractability.

Equation (87) yields a steady state value for $\bar{R} = \frac{1}{\beta}$. Combine (88) with (86) to get the following steady-state relationship for digital currency holdings of banked:

$$\bar{DC^h} = \frac{\eta^h_{DC}\bar{C^h}}{1-\beta R^{DC}} \tag{106}$$

One can observe that the resulting steady-state relationship is restrictive in terms of values of R^{DC}. As the value of R^{DC} tends to $\frac{1}{\beta}$, the value of $\bar{DC^h}$ tends to infinity. R^{DC} is thus bounded by $\frac{1}{\beta} = \bar{R}$.

Using condition (106) and budget constrain of the BHH yields the following expression for the digital currency holdings of the banked:

$$\bar{DC^h} = \frac{\eta^h_{DC}\bar{T^h}}{(1-\beta R^{DC}) - \eta^0_{DC}(R^{DC}-1)} \tag{107}$$

We constrain digital currency holdings to be non-negative. We find the values for R^{DC} that guarantee non-negativity of $\bar{DC^h}$. With numerator greater than zero by definition, we look at the denominator and require it to be non-negative, which yields:

$$R^{DC} < 1 + \frac{1-\beta}{\beta + \eta^h_{DC}} \tag{108}$$

This condition determines the upper bound for R^{DC}. This condition implies that the term spread $\bar{R} - \bar{R^{DC}}$ is always positive, assuming $\eta^h_{DC} > 0$ and $\beta < 1$, since:

$$1 + \frac{1-\beta}{\beta + \eta^h_{DC}} < \frac{1}{\beta} \implies 0 < \eta^h_{DC}(1-\beta) \tag{109}$$

A.3 Solution to model with banking and finance sector

A.3.1 The household problem

The FOCs for consumption, labor supply, deposits, digital currencies, and purchases of equity from the BHH problem are:

$$\lambda_t^h = (C_t^h)^{-\sigma}, \tag{110}$$

$$\lambda_t^h w_t = \zeta_0 (L_t^h)^{\zeta} \tag{111}$$

$$1 = \Lambda_{t,t+1}^h \frac{R_t}{\pi_{t+1}}, \tag{112}$$

$$1 = \frac{\eta_{DC}^h}{\lambda_t^h DC_t^h} + \Lambda_{t,t+1}^h \frac{R_t^{DC}}{\pi_{t+1}}, \tag{113}$$

$$1 = \Lambda_{t,t+1}^h \left[\frac{z_{t+1}^k + (1-\delta)Q_{t+1}}{Q_t + \varkappa^h \Gamma \frac{K_t^h}{K_t}} \right], \tag{114}$$

where λ_t^h is the Lagrangian multiplier from the BHH problem and the household SDF is:

$$\Lambda_{t,t+1}^h = \beta \mathbb{E}_t \frac{\lambda_{t+1}^h}{\lambda_t^h}.$$

A.3.2 Rewriting the banker's problem

To setup the problem of the banker as in Section 3.3.2 (Equations (37) and (38)), first iterate the banker's flow of funds constraint (34) forward by one period, and then divide through by n_t to yield:

$$\frac{n_{t+1}}{n_t} = \frac{\left(z_{t+1}^k + (1-\delta)Q_{t+1}\right)}{Q_t} \frac{Q_t k_t^b}{n_t} - \frac{R_t}{\pi_{t+1}} \frac{d_t}{n_t} - \frac{R_t^{DC}}{\pi_{t+1}} \frac{dc_t}{n_t}.$$

Rearrange the balance sheet constraint (32) and use the fact that $dc_t/n_t = x_t \phi_t$, to yield the following:

$$\frac{d_t}{n_t} = \left(1 + \frac{\varkappa^b}{2} x_t^2\right) \phi_t - x_t \phi_t - 1.$$

Substitute this value for d_t/n_t into the expression for n_{t+1}/n_t, and we get:

$$\frac{n_{t+1}}{n_t} = \left(\frac{z_{t+1}^k + (1-\delta)Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}} \right) \phi_t + \left(1 - \frac{\varkappa^b}{2} x_t^2 \phi_t\right) \frac{R_t}{\pi_{t+1}} + \left(\frac{R_t}{\pi_{t+1}} - \frac{R_t^{DC}}{\pi_{t+1}} \right) x_t \phi_t.$$

Substituting this expression into (35), yields the following:

$$
\begin{aligned}
\psi_t &= \mathbb{E}_t \left[\Lambda^h_{t,t+1}(1-\sigma_b+\sigma_b\psi_{t+1}) \left\{ \begin{array}{c} \left(\frac{z^k_{t+1}+(1-\delta)Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}} \right) \phi_t \\ + \left(1 - \frac{\varkappa^b}{2} x_t^2 \phi_t \right) \frac{R_t}{\pi_{t+1}} \\ + \left[\frac{R_t}{\pi_{t+1}} - \frac{R^{DC}_t}{\pi_{t+1}} \right] x_t \phi_t \end{array} \right\} \right] \\
&= \mu_t \phi_t + \left(1 - \frac{\varkappa^b}{2} x_t^2 \phi_t \right) \upsilon_t + \mu_t^{DC} x_t \phi_t,
\end{aligned}
$$

which is (37) in the text.

A.3.3 Solving the banker's problem

With $\{\mu_t, \mu_t^{DC}\} > 0$, the banker's incentive compatibility constraint binds with equality, and so we can write the Lagrangian as:

$$
\mathcal{L} = \psi_t + \lambda_t(\psi_t - \Theta(x_t)\phi_t),
$$

where λ_t is the Lagrangian multiplier. The FOCs are:

$$
(1+\lambda_t) \left[\mu_t + \mu_t^{DC} x_t - \frac{\varkappa^b}{2} x_t^2 \upsilon_t \right] = \lambda_t \Theta(x_t), \tag{115}
$$

$$
(1+\lambda_t) \left[\mu_t^{DC} + \varkappa^b x_t \upsilon_t \right] = \theta \lambda_t \Theta(x_t), \tag{116}
$$

$$
\psi_t = \phi_t \Theta(x_t). \tag{117}
$$

Substitute (117) into the banker's objective function to yield:

$$
\phi_t = \frac{\upsilon_t}{\Theta(x_t) - \mu_t - \mu_t^{DC} x_t + \frac{\varkappa^b}{2} x_t^2 \upsilon_t}, \tag{118}
$$

which is (39) in the text.

A.3.4 Full set of model conditions

BANKED HOUSEHOLDS

$$\lambda_t^h = (C_t^h)^{-\sigma}, \tag{119}$$

$$\lambda_t^h w_t = \zeta_0 (L_t^h)^{\zeta}, \tag{120}$$

$$1 = \Lambda_{t,t+1}^h \frac{R_t}{\pi_{t+1}}, \tag{121}$$

$$1 = \frac{\eta_{DC}^h}{\lambda_t^h DC_t^h} + \Lambda_{t,t+1}^h \frac{R_t^{DC}}{\pi_{t+1}} - \phi_{DC}^h (DC_t^h - \bar{DC}^h), \tag{122}$$

$$1 = \Lambda_{t,t+1}^h \left[\frac{z_{t+1}^k + (1-\delta) Q_{t+1}}{Q_t + \varkappa^h \Gamma \frac{K_t^h}{K_t}} \right], \tag{123}$$

where λ_t^h is the Lagrangian multiplier from the BHH problem and the household SDF is:

$$\Lambda_{t,t+1}^h = \beta \mathbb{E}_t \frac{\lambda_{t+1}^h}{\lambda_t^h}.$$

UHH

Auxiliary variable G_{t+1} derivative wrt M_t

$$G'_{M_t} = \left(\frac{M_{t-1}}{\pi_t} \right)^{\gamma - 1} (DC_t^u)^{1-\gamma} \tag{124}$$

Auxiliary variable G_t derivative wrt DC_t

$$G'_{DC_t^u} = \frac{1}{\gamma}(1-\gamma) \left(\frac{M_{t-1}}{\pi_t} \right)^{\gamma} (DC_t^u)^{-\gamma} \tag{125}$$

Marginal utility of consumption

$$(C_t^u)^{-\sigma} = \lambda_t^u + \mu_t^u \tag{126}$$

Euler equation for real money balances

$$\beta \left[\frac{\lambda_{t+1}^u}{\pi_{t+1}} + \mu_{t+1} G'_{t+1}(M_t) \right] = \lambda_t^u (1 + \phi_M^u (M_t - \bar{M})) \tag{127}$$

Euler equation for digital currency holdings

$$\beta \frac{\lambda_{t+1}^u R_t^{DC}}{\pi_{t+1}} + \mu_t^u G'_t(DC_t^u) = \lambda_t^u \tag{128}$$

CIA constraint

$$C_t^u \leq \frac{1}{\gamma} \left(\frac{M_{t-1}}{\pi_t} \right)^{\gamma} (DC_t^u)^{1-\gamma}. \tag{129}$$

CAPITAL GOODS PRODUCERS

Investment optimality condition

$$Q_t = 1 + \frac{\kappa_I}{2}\left(\frac{I_t}{\bar{I}} - 1\right)^2 - \frac{\kappa_I}{\bar{I}}\left(\frac{I_t}{\bar{I}} - 1\right) \tag{130}$$

Law of motion for capital

$$K_t = I_t + (1-\delta)K_{t-1} \tag{131}$$

BANKERS' PROBLEM

Maximum leverage ratio:

$$\phi_t = \frac{\upsilon_t}{\Theta(x_t) - \mu_t - \mu_t^{DC} x_t + \frac{\varkappa^b}{2} x_t^2 \upsilon_t} \tag{132}$$

Tobin's Q:

$$\psi_t = \Theta(x_t)\phi_t \tag{133}$$

Banker cost variables and SDF:

$$\mu_t = \mathbb{E}_t\left[\Omega_{t,t+1}\left\{\frac{z_{t+1}^k + (1-\delta)Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}}\right\}\right] \tag{134}$$

$$\mu_t^{DC} = \mathbb{E}_t\left[\Omega_{t,t+1}\left\{\frac{R_t}{\pi_{t+1}} - \frac{R_t^{DC}}{\pi_{t+1}}\right\}\right] \tag{135}$$

$$\upsilon_t = \mathbb{E}_t\left[\Omega_{t,t+1}\frac{R_t}{\pi_{t+1}}\right] \tag{136}$$

$$\Omega_{t,t+1} = \Lambda_{t,t+1}^h(1 - \sigma_b + \sigma_b\psi_{t+1}) \tag{137}$$

AGGREGATION

Aggregate quantities:

$$C_t = \Gamma C_t^h + (1-\Gamma)C_t^u \tag{138}$$

$$L_t = \Gamma L_t^h + (1-\Gamma)L_t^u \tag{139}$$

$$DC_t = \Gamma DC_t^h + (1-\Gamma)DC_t^u \tag{140}$$

$$K_t = \Gamma(K_t^h + K_t^b) \tag{141}$$

Resource constraint:

$$Y_t = C_t + \left[1 + \frac{\kappa_I}{2}\left(\frac{I_t}{\bar{I}} - 1\right)^2\right] I_t + \Gamma\chi_t^h + (1-\Gamma)\chi_t^M + \Gamma\chi_t^b \tag{142}$$

Net worth of banks:

$$\begin{aligned} N_t = \sigma_b \left[(z_t^k + (1-\delta)Q_t)K_{t-1}^b - \frac{R_{t-1}}{\pi_t} D_{t-1} - \frac{R_{t-1}^{DC}}{\pi_t} \frac{DC_{t-1}}{\Gamma} \right] \\ + \gamma_b (z_t^k + (1-\delta)Q_t) \frac{K_{t-1}}{\Gamma}, \end{aligned} \tag{143}$$

Balance sheet variables:

$$Q_t K_t^b = \phi_t N_t, \tag{144}$$

$$Q_t K_t^b \left(1 + \frac{\varkappa^b}{2} x_t^2\right) = D_t + \frac{DC_t}{\Gamma} + N_t, \tag{145}$$

$$x_t = \frac{DC_t}{Q_t \Gamma K_t^b}. \tag{146}$$

A.4 Solution to the TANK model with CBDCs

A.4.1 The steady state

In the non-stochastic steady state, we have the following:

$$\begin{aligned} \bar{Q} &= 1, \\ \bar{\pi} &= 1, \\ \bar{R} &= \frac{1}{\beta}, \\ \bar{R}^{DC} &= \frac{1}{\beta} - \bar{\Delta}^{DC}. \end{aligned}$$

We define the discounted spreads on equity and DC as:

$$s = \beta[\bar{z}^k + (1-\delta)] - 1, \tag{147}$$

$$s^{DC} = 1 - \beta \bar{R}^{DC}, \tag{148}$$

which we consider to be endogenous and exogenous, respectively.

From the BHH's FOC wrt to equity, (123), we have:

$$\begin{aligned} 1 &= \beta \left[\frac{\bar{z} + (1-\delta)}{1 + \varkappa^h \Gamma \frac{\bar{K}^h}{\bar{K}}} \right] \\ 1 + \varkappa^h \Gamma \frac{\bar{K}^h}{\bar{K}} &= \beta \left[\bar{z} + (1-\delta) \right] \\ \Gamma \frac{\bar{K}^h}{\bar{K}} &= \frac{s}{\varkappa^h}. \end{aligned} \tag{149}$$

Additionally, in steady state we have:

$$\begin{aligned}
\bar{\Omega} &= \beta(1-\sigma+\sigma\bar{\psi}), \\
\bar{\upsilon} &= \frac{\bar{\Omega}}{\beta}, \\
\bar{\mu} &= \bar{\Omega}\left[\bar{z} + (1-\delta) - \frac{1}{\beta}\right], \\
\bar{\mu}^{DC} &= \bar{\Omega}\left[\frac{1}{\beta} - \bar{R}^{DC}\right],
\end{aligned}$$

and so, using (147) and (148), we can write:

$$\begin{aligned}
\frac{\bar{\mu}}{\bar{\upsilon}} &= s, \\
\frac{\bar{\mu}^{DC}}{\bar{\upsilon}} &= s^{DC}.
\end{aligned}$$

Next, define J as:

$$J = \frac{n_{t+1}}{n_t} = \left[\bar{z}^k + (1-\delta)\right]\frac{\bar{K}^b}{\bar{N}} - \bar{R}\frac{\bar{D}}{\bar{N}} - \bar{R}^{DC}\frac{\bar{DC}}{\Gamma\bar{N}},$$

and use the following:

$$\begin{aligned}
\frac{\bar{D}}{\bar{N}} &= \left(1 + \frac{\varkappa^b}{2}\bar{x}^2\right)\bar{\phi} - \bar{x}\bar{\phi} - 1, \\
\bar{\phi} &= \frac{\bar{K}^b}{\bar{N}}, \\
\frac{\bar{DC}}{\Gamma\bar{N}} &= \bar{\phi}\bar{x},
\end{aligned}$$

to write J as:

$$\begin{aligned}
J &= (\bar{z}^k + (1-\delta) - \bar{R})\bar{\phi} + \left(1 - \frac{\varkappa^b}{2}\bar{x}^2\bar{\phi}\right)R + (\bar{R} - \bar{R}^{DC})\bar{x}\bar{\phi} \\
&= \frac{1}{\beta}\left[p(s, s^{DC})\bar{\phi} + 1\right],
\end{aligned}$$

where

$$p(s, s^{DC}) \equiv s + s^{DC} - \frac{\varkappa^b}{2}\bar{x}^2,$$

is defined as the return premium.

Then, from (49) we have:

$$
\begin{aligned}
\bar{N} &= \sigma_b \left\{ \left[\bar{z}^k + (1-\delta)\right] \bar{K}^b - \bar{R}\bar{D} - \bar{R}^{DC}\frac{\bar{DC}}{\Gamma} \right\} + \gamma_b \left[\bar{z}^k + (1-\delta)\right] \frac{\bar{K}}{\Gamma} \\
\frac{\bar{N}}{\bar{N}} &= \sigma_b \left\{ \left[\bar{z}^k + (1-\delta)\right] \frac{\bar{K}^b}{\bar{N}} - \bar{R}\frac{\bar{D}}{\bar{N}} - \bar{R}^{DC}\frac{\bar{DC}}{\Gamma\bar{N}} \right\} + \frac{\gamma_b}{\bar{N}} \left[\bar{z}^k + (1-\delta)\right] \frac{\bar{K}}{\Gamma} \\
\beta &= \sigma_b \beta J + \frac{\gamma_b}{\bar{N}}\beta \left[\bar{z}^k + (1-\delta)\right] \frac{\bar{K}}{\Gamma} \\
&= \sigma_b \beta J + \frac{\gamma_b \bar{K}^b}{\bar{N}} \left(1 + \varkappa^h \Gamma \frac{\bar{K}^h}{\bar{K}}\right) \frac{\bar{K}}{\Gamma \bar{K}^b} \\
&= \sigma_b \beta J + \gamma_b (1+s)\bar{\phi}\frac{1}{\frac{\Gamma \bar{K}^b}{\bar{K}}} \\
&= \sigma_b \beta J + \gamma_b (1+s)\bar{\phi}\frac{1}{\frac{\bar{K}-\Gamma \bar{K}^h}{\bar{K}}} \\
&= \sigma_b \left[p(s, s^{DC})\bar{\phi} + 1\right] + \gamma_b (1+s)\bar{\phi}\frac{1}{1-\frac{s}{\varkappa^h}} \\
\beta &= \sigma_b + \left[\sigma_b p(s, s^{DC}) + \gamma_b \frac{1+s}{1-\frac{s}{\varkappa^h}}\right]\bar{\phi},
\end{aligned}
$$

or

$$
\bar{\phi} = \frac{\beta - \sigma_b}{\sigma_b p(s, s^{DC}) + \gamma_b \frac{s+1}{1-\frac{s}{\varkappa^h}}}
$$

Equation (35) in steady state gives us:

$$
\begin{aligned}
\bar{\psi} &= \beta(1 - \sigma + \sigma\bar{\psi})L \\
&= \beta L - \beta\sigma_b L + \beta\sigma_b\bar{\psi}L \\
&= \beta(1-\sigma_b)L + \beta\sigma_b\bar{\psi}L \\
&= \frac{\beta(1-\sigma)L}{1-\beta\sigma_b L} \\
&= \frac{(1-\sigma_b)\left[p(s, s^{DC})\bar{\phi} + 1\right]}{1 - \sigma_b\left[p(s, s^{DC})\bar{\phi} + 1\right]} \\
&= \frac{(1-\sigma_b)\left[p(s, s^{DC})\bar{\phi} + 1\right]}{1 - \sigma_b - \sigma_b p(s, s^{DC})\bar{\phi}},
\end{aligned}
$$

and from (117) we have

$$
\bar{\psi} = \Theta(\bar{x})\bar{\phi}.
$$

Combine the expressions for $\bar{\phi}$ and $\bar{\psi}$ to get:

$$\frac{\Theta(\bar{x})(\beta-\sigma_b)}{\sigma_b p(s,s^{DC})+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}}=\frac{(1-\sigma_b)\left[\frac{p(s,s^{DC})(\beta-\sigma_b)}{\sigma_b p(s,s^{DC})+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}}+1\right]}{1-\sigma_b-\sigma_b\left[\frac{p(s,s^{DC})(\beta-\sigma_b)}{\sigma_b p(s,s^{DC})+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}}\right]},$$

then rearrange:

$$\begin{aligned}0&=H(s,s^{DC})\\&=(1-\sigma_b)\left[\beta p(s,s^{DC})+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}\right]\left[\sigma_b p(s,s^{DC})+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}\right]\\&\quad-\Theta(\bar{x})(\beta-\sigma_b)\left[\sigma_b(1-\beta)p(s,s^{DC})+(1-\sigma_b)\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}\right].\end{aligned}$$

We can observe that as $\gamma_b,\theta^b\to 0$,

$$\begin{aligned}H(s,s^{DC})&=(1-\sigma_b)\left[\beta p(s,s^{DC})\right]\left[\sigma_b p(s,s^{DC})\right]\\&\quad-\theta_0^b(\beta-\sigma_b)\left[\sigma_b(1-\beta)p(s,s^{DC})\right]\\\implies p(s,s^{DC})&\to\theta_0^b\frac{(\beta-\sigma_b)(1-\beta)}{(1-\sigma_b)\beta}\end{aligned}$$

Thus, there exists a unique steady state equilibrium with positive spread $s>0$ for a small enough s^{DC} and γ_b.

Given s, we then yield:

$$\bar{z}^k=\frac{1}{\beta}(1+s)-(1-\delta),$$

and we also have:

$$\begin{aligned}\bar{MC}&=\left(\frac{\bar{z}^k}{\alpha}\right)^{\alpha}\left(\frac{\bar{w}}{1-\alpha}\right)^{1-\alpha}=1\\&=\left(\frac{Y}{K}\right)^{\alpha}\left(\frac{Y}{L}\right)^{1-\alpha}\\&=\left(\frac{\bar{Y}}{\bar{K}}\right)^{\alpha}\left(\frac{\bar{z}^k}{\alpha}\right)^{\frac{\alpha(1-\alpha)}{\alpha-1}},\end{aligned}$$

with

$$
\begin{aligned}
\bar{w} &= (1-\alpha)\frac{\bar{Y}}{\bar{L}}, \\
\bar{z}^k &= \alpha\frac{\bar{Y}}{\bar{K}}, \\
\implies \bar{w} &= (1-\alpha)\left(\frac{\bar{z}}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}.
\end{aligned}
$$

Put these together to get:

$$
\begin{aligned}
\frac{\bar{K}}{\bar{Y}} &= \left(\frac{\bar{z}^k}{\alpha}\right)^{\frac{1-\alpha}{\alpha-1}} \\
&= \frac{\alpha}{\bar{z}^k}.
\end{aligned}
$$

From the FOCs of the BHH and UHH problem, we have:

$$
\begin{aligned}
(\bar{C}^h)^{-\sigma}\bar{w} &= \zeta_0(\bar{L}^h)^{\zeta}, \\
(\bar{C}^u)^{-\sigma}\bar{w} &= \zeta_0(\bar{L}^u)^{\zeta}.
\end{aligned}
$$

These imply:

$$
\begin{aligned}
\bar{C}^h &= \left(\frac{\bar{w}}{\zeta_0(\bar{L}^h)^{\zeta}}\right)^{\frac{1}{\sigma}}, \\
\bar{C}^u &= \left(\frac{\bar{w}}{\zeta_0(\bar{L}^u)^{\zeta}}\right)^{\frac{1}{\sigma}},
\end{aligned}
$$

which gives:

$$
\begin{aligned}
\bar{C} &= \Gamma\left(\frac{\bar{w}}{\zeta_0(\bar{L}^h)^{\zeta}}\right)^{\frac{1}{\sigma}} + (1-\Gamma)\left(\frac{\bar{w}}{\zeta_0(\bar{L}^u)^{\zeta}}\right)^{\frac{1}{\sigma}} \\
&= \bar{L}^{-\frac{\zeta}{\sigma}}\left(\frac{\bar{w}}{\zeta_0}\right)^{\frac{1}{\sigma}}.
\end{aligned}
$$

Additionally, we have:

$$
\begin{aligned}
&\frac{\bar{I}}{\bar{K}} = \delta, \\
&\frac{1}{\beta} = \frac{\alpha\frac{\bar{Y}}{\bar{K}} + 1 - \delta}{1 + \varkappa^h\Gamma\frac{\bar{K}^h}{\bar{K}}} \Leftrightarrow \frac{\bar{Y}}{\bar{K}} = \frac{\beta^{-1}(1+s) + \delta - 1}{\alpha},
\end{aligned}
$$

from (149), and:

$$\frac{\bar{I}}{\bar{Y}} = \frac{\bar{I}/\bar{K}}{\bar{Y}/\bar{K}} = \frac{\alpha\delta}{\beta^{-1}(1+s)+\delta-1}.$$

Then, we have the aggregate resource constraint:

$$1 = \frac{\bar{C}}{\bar{Y}} + \frac{\bar{I}}{\bar{Y}} + \Gamma^2 \frac{\varkappa^h}{2}\frac{\bar{K}}{\bar{Y}} + \frac{\varkappa^b}{2}\frac{\bar{DC}}{\bar{Y}}.$$

We now aspire to get a steady-state value for $\frac{DC}{C}$. DC is aggregated as follows

$$DC = \Gamma * DC^h + (1-\Gamma)DC^u \tag{150}$$

We use BHH Euler equation for consumption:

$$1 = \frac{\eta^h_{DC}}{(C^h)^{-\sigma} DC^h} + \frac{1}{\beta} R^{DC} \tag{151}$$

$$(C^h)^{-\sigma} = \frac{\eta^h_{DC}}{DC^h} + \frac{1}{\beta} R^{DC}$$

$$DC^h = \eta^h_{DC}(C^h)^{\sigma} + \frac{1}{\beta} R^{DC}$$

$$C^h = \left[\frac{\left(DC^h - \frac{1}{\beta}R^{DC}\right)}{\eta^h_{DC}}\right]^{\frac{1}{\sigma}}$$

We use standard calibration value for $\sigma = 2$, hence the equation is quadratic in C^h, we further use its positive root, hence

$$C^h = \left[\frac{\left(DC^h - \frac{1}{\beta}R^{DC}\right)}{\eta^h_{DC}}\right]^{\frac{1}{2}}$$

UHH

Auxiliary variable G_{t+1} derivative wrt M_t

$$G'_{M_t} = \left(\frac{M_{t-1}}{\pi_t}\right)^{\gamma-1} (DC^u_t)^{1-\gamma} \tag{152}$$

Auxiliary variable G_t derivative wrt DC_t

$$G'_{DC^u_t} = \frac{1}{\gamma}(1-\gamma)\left(\frac{M_{t-1}}{\pi_t}\right)^{\gamma} (DC^u_t)^{-\gamma} \tag{153}$$

Marginal utility of consumption

$$(C_t^u)^{-\sigma} = \lambda_t^u + \mu_t^u \tag{154}$$

Euler equation for real money balances

$$\beta \left[\frac{\lambda_{t+1}^u}{\pi_{t+1}} + \mu_{t+1} G'_{t+1}(M_t) \right] = \lambda_t^u (1 + \phi_M^u (M_t - \bar{M})) \tag{155}$$

Euler equation for digital currency holdings

$$\beta \frac{\lambda_{t+1}^u R_t^{DC}}{\pi_{t+1}} + \mu_t^u G'_t(DC_t^u) = \lambda_t^u \tag{156}$$

CIA constraint

$$C_t^u \leq \frac{1}{\gamma} \left(\frac{M_{t-1}}{\pi_t} \right)^{\gamma} (DC_t^u)^{1-\gamma}. \tag{157}$$